A DETERMINATION OF THE APEX

OF THE

SOLAR MOTION

ACCORDING TO THE METHOD OF BRAVAIS

BY

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INTRODUCTION.

Nearly all methods for the determination of the apex of the solar motion are based upon the hypothesis that the real proper motions of the stars have no predilection for any one direction. Recent investigations have shown that this hypothesis is no longer tenable. Both KOBOLD and Prof. KAPTEYN [1]) have shown that there can be no doubt about the existence of strongly systematic deviations from a random-distribution. These led the latter to his theory of „star-streams", a theory which was subsequently confirmed by the investigations of EDDINGTON and DYSON [2]), based on independent materials. It seems imperatively necessary therefore to have a new determination of the position of the apex by means of a method which is independent of this hypothesis of the random-distribution of the proper motions.

It is this reason which led me to undertake such a determination by the method of BRAVAIS. The method, published as early as 1843 [3]), is, in principle, quite free from any hypothesis. It is true that we cannot apply it rigorously owing to the fact that several of the quantities contained in the equations, to which we are led, are still unknown. Still I think that, at the present moment

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[1]) Report of the British Association for the Advancement of Science 1905, pag. 257.

[2]) EDDINGTON: Monthly Notices of the Roy. Astron. Society 1906, page 34.
DYSON: Proceedings of the Roy. Society of Edinburgh, sess. 1907—'08, vol. XXVIII, part. III, no. 13.
The material used by EDDINGTON has been treated in another way by Schwarzschild.

[3]) Journal de Mathématiques de Liouville, tome VIII, année 1843.

there is no other known method which is entitled to the same confidence Reasons for this greater confidence will be shown Chapter I.

Along with my aim

1st of determining the position of the apex as free as possible from any supposition about the proper motions

I had another aim in view, *viz.*:

2nd to find out whether the positions of the apex yielded by stars of different galactic latitude were the same or were decidedly different.

CHAPTER I.

DISCUSSION OF THE METHOD OF BRAVAIS.

Short explanation of the method.

BRAVAIS considers the following problem: to determine the sun's motion relative to the centre of gravity of an arbitrary group of stars, the sun itself included. If the universe is assumed to be finite, we may imagine the group chosen in such a way that all the stars are contained in it; and we shall then get the sun's motion relative to the centre of gravity of the universe. BRAVAIS has based the solution of his problem upon a principle of pure mechanics. As the origin of coordinates he takes the sun's position at the moment which is taken as the origin of time; the origin of coordinates is further supposed to be at rest relatively to the centre of gravity of the group. Through this point three fixed perpendicular axes are taken as axes of coordinates. If the components of the velocity of a star with respect to the adopted system of coordinates are called respectively Δx, Δy and Δz and the mass m, then, as an immediate result from the choice of the origin, we have for all stars together:

$$(1) \quad . \quad . \quad . \quad . \quad \Sigma(m\Delta x) = 0. \qquad \Sigma(m\Delta y) = 0. \qquad \Sigma(m\Delta z) = 0.$$

These equations form the basis of the method; after some simple mathematical considerations they are reduced to three equations which are given on page 442 of the paper of BRAVAIS. These express the components of the solar motion as a function of the spherical coordinates of the stars, their distances, masses, observed proper motions and radial peculiar velocities [1]).

Finally BRAVAIS chooses the axes of coordinates in such a way that the positive axis of z points to the north pole, the positive axis of x to the point Aries and the positive axis of y to a point in the equator with a right-ascension of 6 hours. Let us call the rectangular components of the solar velocity ξ, η and ζ; the right-ascension and declination of a star, α and δ; its distance from the sun, ϱ; the observed proper motion of the star in right-ascension and declination, expressed respectively in seconds of time and of arc, μ_α and μ_δ; its

[1]) This name we shall give to the radial velocities relative to a *fixed* sun as origin.

radial peculiar velocity μ_ρ (taken positive if the motion is directed from the sun); the mass, m. If now we exclude the sun from the group of stars of which the centre of gravity is rigidly connected with the origin, the equations on page 442 quoted above may be easily reduced to the following form.

$$\begin{aligned}
&\Sigma m(1 - \cos^2\delta\cos^2\alpha)\xi - \Sigma m\cos^2\delta\sin\alpha\cos\alpha\,.\,\eta - \Sigma m\cos\delta\sin\delta\cos\alpha\,.\,\zeta = \\
&\qquad \Sigma m\varrho(15\sin\alpha\cos\delta\,.\,\mu_\alpha + \sin\delta\cos\alpha\,.\,\mu_\delta) - \Sigma m\cos\delta\cos\alpha\;\mu_\rho. \\
&\Sigma m(1 - \cos^2\delta\sin^2\alpha)\eta - \Sigma m\cos^2\delta\sin\alpha\cos\alpha\,.\,\xi - \Sigma m\cos\delta\sin\delta\sin\alpha\,.\,\zeta = \\
&\qquad \Sigma m\varrho(-15\cos\alpha\cos\delta\,.\,\mu_\alpha + \sin\delta\sin\alpha\,.\,\mu_\delta) - \Sigma m\cos\delta\sin\alpha\,.\,\mu_\rho. \\
&\Sigma m\cos^2\delta\,.\,\zeta - \Sigma m\cos\delta\sin\delta\cos\alpha\,.\,\xi - \Sigma m\cos\delta\sin\delta\sin\alpha\,.\,\eta = \\
&\qquad \Sigma m\varrho(-\cos\delta\mu_\delta) - \Sigma m\sin\delta\,.\,\mu_\rho.
\end{aligned} \tag{2}$$

These equations are rigorous and therefore they determine, free from any hypothesis, the solar motion relative to the adopted origin as a function of the other quantities contained. In order to get his definitive equations [1]) BRAVAIS introduced certain suppositions in regard to the masses, distances and radial velocities of the stars. By so doing he obtained a form somewhat simpler than the equations (2); but they are no longer a pure expression of the fundamental principle. For this reason we shall provisionally retain the equations in the form (2).

Objections to the method of Bravais.

Evidently the equations in the form (2) contain quantities which are unknown or known only to a small extent, *viz*:

1. the masses.
2. the distances.
3. the radial peculiar velocities.

Hence there are difficulties in the way of applying the method of BRAVAIS to the determination of the apex. As we have said just now BRAVAIS extricates himself from these by making certain simple suppositions. As in these suppositions lies BRAVAIS' weak point, a just appreciation of the value of his method requires a thorough examination of the difficulties thus introduced. They may be expressed in the form of the two following questions.

1. In how far can we meet these difficulties? On the answer to this question depends the absolute value of the method.

2. How do the difficulties compare with those arising from the other methods? The masses and radial peculiar velocities do not enter into the equations furnished by the majority of other methods; the distances either do not appear in them or may be eliminated by a slight modification of the method. Hence, owing to the absence of these quantities, the other methods might seem to be

[1]) Page 446 of the paper of BRAVAIS.

superior to the one under discussion. But are these advantages really as great as they seem to be or, perhaps, do the same or equivalent difficulties — though somewhat masked — present themselves in the other methods? Evidently this question belongs under the second head, *i. e.* the relative value of the method of Bravais as compared with the other methods.

First question.

We shall treat separately the masses, distances and radial velocities.

a) *The masses.*

Bravais gives an application of his method to 61 stars with well determined proper motions. Here he takes all the masses to be equal, and they thus disappear from the equations. This supposition is equivalent to the adoption of a new origin of coordinates which is at rest relatively to what we may call the *geometrical* centre of gravity [1]) instead of the *mechanical* one. For if we had started with the equations

$$\Sigma\Delta x = 0 \qquad \Sigma\Delta y = 0 \qquad \Sigma\Delta z = 0 \tag{3}$$

as a basis for the method instead of those of (1) we would have derived equations identical with those of (2), only all the factors m would be replaced by *unity*. Now the equations (3) are rigorous if we take as the origin of coordinates a point which is at rest relatively to the geometrical centre of gravity [2]); and therefore the modified equations obtained by replacing in the equations (2) all the factors m by unity are also rigorous in this case. Thus by putting all the masses equal to unity in the equations (2), we get equations, which yield the correct values of the components of the solar motion, relative to the geometrical centre of gravity of all the stars [3]). If the stars move with uniform velocity in straight lines it is quite indifferent whether we get the sun's motion relative to the geometrical or to the mechanical centre of gravity. For in the present state of science, and presumably for many centuries to come, we have to consider the proper motions of the stars as uniform and rectilinear. With such a motion of the individual stars, the motion of the geometrical centre of gravity has the same property as that of the mechanical centre, *viz* that of being uniform and rectilinear and the two present exactly the same advantage for the present problem. No doubt if afterwards, in the very remote future, the curvature of the real orbits of the stars becomes sensible, the mechanical centre of gravity will be preferable as an origin. In this case however the methods which are based on the hypothesis of a random-distribution are liable to the *same* objection because these also determine the sun's motion relative to the geometrical centre of gravity, as will be shown afterwards.

Howsoever this be, for the present at all events there can be no serious objection against the choice of the geometrical centre of gravity. The supposition

[1]) For the exact definition of this point see Notes to Chapter I. 1).
[2]) See idem.
[3]) The axes of coordinates must be imagined to remain parallel to the directions originally chosen.

about the masses amounts to a change of origin and not to the introduction of a new hypothesis.

b) *The distances.*

The distances are also taken as equal by BRAVAIS. This is equivalent to giving a preponderance to the stars nearest the solar system.

The magnitudes have often been taken as a criterion of the distances of the stars. But the criterion is a very imperfect one because the total quantities of light emitted are very different.

A better criterion is furnished by the proper motions. The distances might be taken to be in inverse ratio to the observed proper motions, though this proceeding may be considered liable to objection. Other things being equal, the observed proper motions will be smaller for the stars, which move in the direction of the apex than for those which move in the opposite direction and as a consequence the distances derived for the first class of stars would systematically exceed those for the second class. This might cause an error in the determination of the apex in the case that the distribution of the peculiar proper motions is in some way systematic. For that reason a somewhat more refined proceeding seems to be desirable We may avoid the difficulty in the following manner.

By means of an approximate position of the apex we can determine for every star the angle p between the direction of the observed proper motion and the great circle towards the ant-apex. Denoting now the angular distance between the star and the apex by λ; we may divide the stars into groups according to the value of sin λ and these again into groups according to the value of p. For each of the so formed groups we may derive the mean value of the proper motions, which we will call $\overline{\mu}_{\lambda, p}$ and divide this by the mean value $\overline{\mu}$ of the proper motions of all the stars. Denoting $\frac{\overline{\mu}_{\lambda, p}}{\overline{\mu}}$ by q we shall have $q = \varphi(\lambda, p)$. This function can therefore be determined empirically. We may, however, also determine it theoretically by making a simple supposition with regard to the distribution of the peculiar proper motions. If we put $f = \frac{1}{q}$ and denote the observed proper motions of the stars by μ, then the quantities $\mu' = f\mu$, which we shall call the reduced proper motions, will be independent of the values and p and λ. From these quantities μ' we may thus consider the effect of the sun's motion on the distribution to have been eliminated. They will afford a better criterion for the distances than the directly observed quantities μ.

Finally in connection with these quantities μ' the magnitudes may be used for the estimation of the distances. Tables have been derived empirically [1]) by Prof. KAPTEYN, which give the mean parallax of stars of determined proper motion and magnitude. If now instead of the proper motions μ, we take the reduced proper motions μ' as the argument, we may take from these tables values

[1]) Publ. 8 of the Astronomical Laboratory of Groningen.

for the parallaxes and hence for the distances, which, substituted in the equations of BRAVAIS, are sufficient for a determination of the *position* of the apex

In this way the difficulties of the distances may be avoided to a great extent.

c) *The radial velocities.*

Of these quantities nothing was yet known in the time of BRAVAIS. There thus was no other way but to neglect them altogether. Evidently this proceeding is equivalent to the introduction of a hypothesis. Considering the origin of coordinates to be at rest relatively to the geometrical centre of gravity so that the masses disappear from our equations we shall have (see equations 2):

$$\Sigma \cos\delta \cos\alpha \,.\, \mu_p = 0,$$
$$\Sigma \cos\delta \sin\alpha \,.\, \mu_p = 0,$$
$$\Sigma \sin\delta \,\mu_p = 0,$$

in which the sums are extended over the whole material used for the determination of the apex. These equations may be enunciated in the following way:

the sums of the projections of the radial peculiar velocities of all the stars upon the three axes of coordinates are zero, or:

the radial peculiar velocities of all the stars transferred to the geometrical centre of gravity and treated as forces, will be in equilibrium.

We shall call this supposition the hypothesis of BRAVAIS. Since at the present time with the aid of spectroscopy, something more of the radial velocities has become known, it might seem possible to get rid of a part of the uncertainty introduced by this hypothesis, introducing the values furnished by the spectroscope. The number of published spectroscopic velocities however is still too small at the present time for expecting an improvement by this. Moreover there exists another and more radical difficulty which is due to the fact that what the spectroscope really determines is the radial motion of a star relative to the moving sun. From this relative motion the radial *peculiar* velocity μ_p might be obtained if we had some independant method of determining the sun's linear velocity. In fact, however, this linear velocity is itself derived from the spectroscopic observations on the hypothesis that the radial peculiar velocities are in equilibrium. But this hypothesis is identically the same as that of BRAVAIS. Therefore by substituting in the formulae (2) for the radial peculiar velocities, the values derived from the observed spectroscopic velocities, no improvement is to be expected.

How this difficulty might be overcome will be shown later. For the present there seems to be no better way than simply to neglect the terms with μ_p, thus adopting the hypothesis of BRAVAIS. This hypothesis might introduce an error in the position derived for the apex if some such supposition as that of HERSCHEL, who assumes a condensation of the stellar system, were correct, especially if the condensation were directed towards a point not too far distant from the Sun. But, as has been already remarked by BRAVAIS even in such a case the error will vanish if opposite areas of the sky contain equal numbers of stars.

If the distribution of the stars selected for the determination of the apex does not quite fulfill the condition stated above, we may meet the difficulty by a suitable choice of the weights. For if we take care that the terms furnished by zones of the same area and equal but opposite declinations appear in the definitive equations with the same weights, the effect will be the same, with respect to the radial velocities, as if they contained an equal number of stars.

If this precaution be taken, then the hypothesis of BRAVAIS will be satisfied not only in the case here considered of a condensation of the whole system but in a number of other cases besides.

It is satisfied certainly for a sufficient number of stars if the directions of the peculiar proper motions are supposed to be distributed at random. But other distributions of the velocities also are compatible with the hypothesis of BRAVAIS.

If the theory of the star-streams referred to in the introduction gives a correct representation of the real distribution of cosmic velocities there must exist an equal preference for motions in two opposite directions. We shall suppose that an equal preference for both directions exists in every part of space, as will be the case if the two streams are well intermingled everywhere. If this condition is fulfilled the radial velocities will neutralize each other in every individual region. So the theory of the star-streams which may be considered the only supposition in regard to the distribution of the real velocities that is based directly on experience does not contradict the hypothesis of BRAVAIS.

SCHÖNFELD has investigated another distribution of the velocities. He has considered the question whether there is a systematic rotation about an axis perpendicular to the plane of the galaxy. If this were the case we might assume this axis to pass through the geometrical centre of gravity, as this is supposed to be at rest. This however can not lie far from the sun as a consequence of the adopted distribution of the stars; therefore the supposed rotation has no influence upon the radial velocities, and in this case also there will be a compensation of the terms neglected by BRAVAIS.

If however the distribution of stellar velocities in space is such that the radial peculiar velocities of all the stars do not neutralize each other, no precaution whatever can prevent the apex determined from purely astronomical proper motions to differ from the true one. But even the introduction of the radial velocities derived from spectroscopic results will give no improvement as we have shown above.

However it will be shown at the end of this chapter that spectroscopic results may be combined in another way with astronomical ones to determine the apex free from any hypothesis about the radial velocities. This will require a fairly large number of suitable spectroscopic velocities.

Second question.

Difficulties arising from BRAVAIS' method compared with those presented by the other methods.

a) *The masses.*

The masses appear in the equations when we adopt as origin a point rigidly connected with the mechanical centre of gravity. By taking as the origin a point holding the same relation to the geometrical centre of gravity they are eliminated. In the other methods generally it is not stated definitely which point is to be taken as the origin; but it may easily be proved that in the methods which are based on the hypothesis of the random-distribution as those of AIRY, ARGELANDER etc. the *geometrical* centre of gravity must really be considered as the origin.

We may assert that, if the hypothesis of the random-distribution is correct, this is only the case relatively to a point of which the motion is perfectly determined. For if we suppose it to be true relatively to a point P as origin, this will be no longer the case with reference to another point Q, which moves relatively to P. For to get the motions relatively to the new origin Q, we must form for every star the resultant of the original proper motion and a velocity equal and parallel to that of P relatively to Q. By doing so the random-distribution will be destroyed. In adopting the hypothesis of the random-distribution we must therefore take the point P or a point rigidly connected with P as origin. Now the origin P, as a consequence of the supposition made, has the property that all the proper motions transferred to it, will neutralize each other. But this is precisely a property of the geometrical centre of gravity and it is not possessed by any other point which moves relatively to it. The point P must therefore be rigidly connected with the geometrical centre of gravity.

We may therefore assert that the hypothesis of the random-distribution requires the assumption of the geometrical centre of gravity or a point rigidly connected with it as an origin. Therefore the methods which are based on this hypothesis can only determine the motion of the sun relative to the geometrical centre of gravity. The non-appearance of the masses in the equations to which these methods lead, cannot therefore be considered as a real advantage of these methods over that of BRAVAIS.

In what follows we shall always suppose, unless the contrary is stated, that the origin is rigidly connected with the *geometrical centre of gravity*.

b) *The distances.*

Among the methods which rest on the hypothesis of the random-distribution, only that of AIRY [1]) contains the distances in the equations. We may however transform this method in such a way that we get two sets of equations from one of which the distances disappear [2]). Also the method of ARGELANDER [3]) and that of Prof. KAPTEYN [4]) lead to equations from which the distances are

[1]) Memoirs of the Royal Astronomical Society of London, vol. XXVIII, page 143 etc.
[2]) Astron. Nachr. No. 3721—'22, Meth. III, page 9.
[3]) Astron. Nachr. No. 363.
[4]) Proceedings of the meetings of the Akademy of Sciences at Amsterdam, Vol. II, page 353 etc.

absent. Indeed the hypothesis of the random distribution affords a principle by which it is possible to determine the position of the apex purely by means of considerations of symmetry without taking the distances into account. But another difficulty presents itself generally, namely that of assigning the weights with which the proper motions must be combined. As Prof. KAPTEYN has shown this difficulty may be removed by grouping the stars according to the amount of the proper motions and combining the results obtained from the individual groups by weights in accordance with the mean errors in the results. This proceeding supposes that the values of the apex obtained from groups of stars at different distances from the solar system must be really identical, and this in its turn may be considered to be a consequence of the hypothesis of the random-distribution. We see therefore that in the derivation of the apex according to one of these methods this hypothesis plays a double part. The influence of it is a *primary* one, in so far as the equations used are themselves expressions of the hypothesis of the random-distribution. It has a more subordinate *secondary* influence in the choice of the weights.

On the other hand we have seen that we may derive estimated values for the distances, which are sufficient for the determination of the position of the apex by the method of BRAVAIS. These estimated distances are based for a large part on empirical data, though in deriving them we cannot entirely avoid introducing some hypothetical elements. The hypothetical elements introduced in this way, however have only a *secondary* influence upon the position of the apex, an influence of the same order as the choice of the weights in the other methods.

c). *The radial velocities.*

As we mentioned already, BRAVAIS, by neglecting the terms depending on the radial velocities introduces the following hypothesis:

Hypothesis B. The resultant of the radial peculiar velocities of all the stars relatively to the geometrical centre of gravity is zero.

We shall now compare this hypothesis with that underlying the other methods. For that purpose we shall denote the hypothesis of the random-distribution by H. As the methods which rest on the hypothesis H take into account only the tangential components of the peculiar velocities, we may here consider the hypothesis H to be relative to these components only. Now the hypothesis B may be expressed in a somewhat different form so that it is more easily comparable with H.

Since the origin is at rest relatively to the geometrical centre of gravity, we have rigorously:

the resultant of the *total* peculiar velocities of all the stars is zero.

Evidently we may put this in another way, *viz*:

the resultants of the tangential and of the radial peculiar proper motions are equal and oppositely directed.

Combining this property with the hypothesis of BRAVAIS, we get:

(C) the resultant of the *tangential* proper motions of all the stars is zero.

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BRAVAIS' hypothesis thus may be put perfectly as well in the form (C) as in the form (B). In our comparison with other methods we will adopt the form (C).

The hypothesis H supposes that for every part of space or of the sky, which contains a sufficient number of stars, separately the condition expressed by (C) is fulfilled. But it involves more than this. It supposes that there is a radial symmetry in the distribution of the peculiar motions. We thus see that the hypothesis H really contains (C) as a part.

Among the methods, based on the hypothesis H, that of Prof. KAPTEYN does not require the absolute correctness of the suppositions implied by the hypothesis H. For as it determines for every point of the sphere the direction of the apex as that of the resultant of the proper motions, it will not be necessary for the application of this method that a radial symmetry in the distribution of the proper motions exists. If for example there is an equal preference for two opposite directions in every part of the sky the position of the apex furnished by this method would still be correct. The other methods however which are based on the hypothesis H cannot be applied without reserve in this case. They presuppose that in the individual parts of the sky generally no large systematic deviations from the radial symmetry exist. For the method of BRAVAIS it is quite indifferent whether they do exist or not, if only all the tangential peculiar proper motions transferred to a single point neutralize each other. This may be considered as a considerable advantage of the method of BRAVAIS over the other methods.

The combination of the astronomical and spectroscopic velocities.

The method of Bravais would not lead to a correct result in the case that the tangential peculiar velocities have a resultant different from *zero*, consequently the radial peculiar velocities an equal resultant in the opposite direction. In this case however we could still obtain the *true* values for the components of the solar motion relative to the geometrical centre of gravity if only a sufficient number of spectroscopic velocities were available [1]). We would then have to combine in the proper way the results obtained from the astromical proper motions with those obtained from the radial velocities. — How the combination must be made will appear from the following consideration.

The equations of BRAVAIS, definitively used for the determination of the apex, are of the following form (see the beginning of Chapter II):

$$\Sigma(1-\cos^2\delta\cos^2\alpha)\xi - \Sigma\cos^2\delta\sin\alpha\cos\alpha\,.\,\eta - \Sigma\sin\delta\cos\delta\cos\alpha\,.\,\zeta = \\ = \Sigma\varrho(15\cos\delta\sin\alpha\,.\,\mu_\alpha + \sin\delta\cos\alpha\,.\,\mu_\delta).$$

$$(4)\quad \Sigma(1-\cos^2\delta\sin^2\alpha)\eta - \Sigma\cos^2\delta\sin\alpha\cos\alpha\,.\,\xi - \Sigma\sin\delta\cos\delta\sin\alpha\,.\,\zeta = \\ = \Sigma\varrho(-15\cos\delta\cos\alpha\,.\,\mu_\alpha + \sin\delta\sin\alpha\,.\,\mu_\delta).$$

$$\Sigma\cos^2\delta\,.\,\zeta - \Sigma\sin\delta\cos\delta\cos\alpha\,.\,\xi - \Sigma\sin\delta\cos\delta\sin\alpha\,.\,\eta = \Sigma\varrho(-\cos\delta\,.\,\mu_\delta).$$

[1]) Of course we will suppose here, that the *distances* would introduce no difficulty in the determination of the components of the solar velocity from the astronomical proper motions.

These equations may be obtained by starting from the fundamental-equations (3) [1]).

In developing the solution we have only to neglect the *radial* peculiar velocities.

Now, in order to get a system of equations analogous to these, which will determine the components ξ, η and ζ from the observed spectroscopic proper motions v, we have to start from the same fundamental equations, neglecting now the *tangential* peculiar proper motions. If we do so, we get the following set of equations [2])

$$\begin{aligned}
&\Sigma \cos^2\delta \cos^2\alpha \,.\, \xi + \Sigma \cos^2\delta \sin\alpha \cos\alpha \,.\, \eta + \Sigma \sin\delta \cos\delta \cos\alpha \,.\, \zeta = -\Sigma v \cos\delta \cos\alpha \\
&\Sigma \cos^2\delta \sin^2\alpha \,.\, \eta + \Sigma \cos^2\delta \sin\alpha \cos\alpha \,.\, \xi + \Sigma \sin\delta \cos\delta \sin\alpha \,.\, \zeta = -\Sigma v \cos\delta \sin\alpha. \\
&\Sigma \sin^2\delta \,.\, \zeta + \Sigma \sin\delta \cos\delta \cos\alpha \,.\, \xi + \Sigma \sin\delta \cos\delta \sin\alpha \,.\, \eta = -\Sigma v \sin\delta.
\end{aligned} \tag{5}$$

These equations are identical with the normal-equations belonging to the equations of condition used by CAMPBELL for his preliminary determination of the solar motion from a group of 280 stars [3]).

If now we imagine the equations (4) and (5) to be relative to the same system of stars of which we denote the number by N, we get by adding the corresponding equations of the two systems:

$$\begin{aligned}
N\xi &= \Sigma[\varrho(15\cos\delta\sin\alpha \,.\, \mu_\alpha + \sin\delta\cos\alpha \,.\, \mu_\delta) - v\cos\delta\cos\alpha]. \\
N\eta &= \Sigma[\varrho(-15\cos\delta\cos\alpha \,.\, \mu_\alpha + \sin\delta\sin\alpha \,.\, \mu_\delta) - v\cos\delta\sin\alpha]. \\
N\zeta &= \Sigma[\varrho(-\cos\delta \,.\, \mu_\delta) - v\sin\delta].
\end{aligned} \tag{6}$$

If we call the projections on the axes of x, y and z of the total linear velocities of the stars in space relative to the *moving* sun δx, δy and δz, then the second members are precisely equal to the sums of these quantities with the negative sign. We may therefore write the equations in the form:

$$\Sigma(\delta x + \xi) = 0. \qquad \Sigma(\delta y + \eta) = 0. \qquad \Sigma(\delta z + \zeta) = 0 \tag{7}$$

But these equations are identical with the equations (3), from which the rigorous equations of Bravais [4]) are derived. By adding therefore the equations (4) and (5) in the proper manner the exact equations for the determination of ξ, η and ζ are formed.

If therefore for a group of stars the spectroscopic as well as the astronomical proper motions were known, we could get from these the true values of the components of the solar motion relative to the geometrical centre of gravity of the group by adding the equations (4) and (5) or more simply by taking directly the equations (6). We will however consider the more general case that the astronomical proper motions are known for a group, which we call M_1, the spectroscopic proper motions for a group M_2, which wholly or partially consists of other stars. Let us call the components of the solar motion as derived from the first

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[1]) See page 5.
[2]) For the proof see Note to Chapter I 2).
[3]) Astrophysical Journal, vol XIII, page 83 etc.
[4]) With respect of course to the *geometrical* centre of gravity.

group ξ_1, η_1 and ζ_1, those from the second group ξ_2, η_2 and ζ_2. By combining these two groups of quantities we can derive the *correct* values for ξ, η and ζ only in the case that both groups M_1 and M_2 may be considered to be fair samples of one and the same group M. If this is *not* the case, the two sets of values will be relative to different points as origin. The relative motion of these two points will be unknown. The two sets of values cannot therefore be properly combined. At the present time spectroscopic proper motions are only known for a group of the brightest stars; astronomical proper motions for a much larger group containing a considerable number of much fainter ones. A combination of the components obtained from the two groups will not therefore be of any particular value at the present time unless we rest satisfied with a determination of the solar motion relative to the very bright stars only.

As soon however as the spectroscopic velocities will be known for a group M_2 containing a sufficient number of stars among which the different magnitudes are represented *nearly in the same proportion* as they are in the group M_1, the case will be different.

The way in which the results obtained separately from the two groups must then be combined, in order to give the most reliable result, takes a particularly simple form if our stars are fairly evenly distributed over the sky. For the two groups M_1 and M_2 may now be considered as fair samples of one and the same more extensive group M of which the stars are distributed evenly over the sky. For such a group the coefficients in the equations (4) and (5) will all vanish except that of ξ in the first, η in the second and ζ in the third equation of each set. These coefficients may be easily found by a simple integration over the sky. Denoting the number of stars of the group M by N, these coefficients become all equal to $\frac{2}{3}N$ in the equations (4) and equal to $\frac{1}{3}N$ in the equations (5).

If now we put:

$$\begin{aligned}
&\Sigma\varrho \quad (15\cos\delta\sin\alpha \,.\, \mu_\alpha + \sin\delta\cos\alpha \,.\, \mu_\delta) = Np_x \\
&\Sigma\varrho\,(-15\cos\delta\cos\alpha \,.\, \mu_\alpha + \sin\delta\sin\alpha \,.\, \mu_\delta) = Np_y \\
&\Sigma\varrho\,(-\cos\delta \,.\, \mu_\delta) = Np_z \\
&\Sigma - v\cos\delta\cos\alpha = Nq_x \\
&\Sigma - v\cos\delta\sin\alpha = Nq_y \\
&\Sigma - v\sin\delta = Nq_z
\end{aligned}$$

then the two sets of equations (4) and (5) assume the following forms:

$$\begin{aligned}
&(4a) \quad \tfrac{2}{3}N\xi_1 = Np_x \qquad \tfrac{2}{3}N\eta_1 = Np_y \qquad \tfrac{2}{3}N\zeta_1 = Np_z \\
&(5a) \quad \tfrac{1}{3}N\xi_2 = Nq_x \qquad \tfrac{1}{3}N\eta_2 = Nq_y \qquad \tfrac{1}{3}N\zeta_2 = Nq_z
\end{aligned}$$

whereas the equations (6) which give the exact values of the components may be written:

$$(6a) \quad N\xi = N(p_x + q_x) \qquad N\eta = N(p_y + q_y) \qquad N\zeta = N(p_z + q_z).$$

Eliminating now the quantities p_x, p_y, p_z, q_x, q_y and q_z from these 9 equations we get:

$$(8)\quad \ldots\ldots\ldots \begin{cases} \xi = \frac{2}{3}\xi_1 + \frac{1}{3}\xi_2 \\ \eta = \frac{2}{3}\eta_1 + \frac{1}{3}\eta_2 \\ \zeta = \frac{2}{3}\zeta_1 + \frac{1}{3}\zeta_2. \end{cases}$$

The assumed uniform distribution of the group M over the sky is not the most representative one. For if our data extend to magnitude m it centainly seems more satisfactory to get the sun's motion referred to the geometrical centre of the *whole* of the stars down to that magnitude. In order to represent fairly the system of all the stars down to the magnitude m, the groups M_1 and M_2 (consequently also M), must satisfy the following condition: the number of stars of different magnitude and galactic latitude contained in our groups must be proportional to the corresponding numbers in the sky. They therefore must approximately satisfy the two conditions:

1°. The distribution must be symmetrical relatively to the plane of the galaxy.

2°. The stars must be distributed equally over the different galactic longitudes.

If the actual M_1 and M_2 do not quite satisfy these conditions we may improve the results by a suitable choice of the weights.

If now we take the plane of the galaxy as plane of XY, the equations (4), (5) and (6) will subsist, only the quantities ξ, η and ζ will now represent galactic components. We will denote them by X, Y and Z. The coordinates α and δ must be replaced by the galactic longitude and latitude l and b; μ_α and μ_δ by the corresponding galactic components of proper motion. In the second members quantities which we will call P_i and Q_i will take the place of the p_i and q_i. As a consequence of the uniform distribution over the different galactic longitudes, all coefficients of X, Y and Z, except that of X in the first, Y in the second and Z in the third equation, will vanish. If again we denote by the index 1 the values of the components of the solar motion derived from the astronomical, by 2 those from the spectroscopic velocities and without an index those found from the total velocities of the stars of group M, we get the following equations (N = number of stars):

$$(4b)\quad \Sigma(1-\cos^2 b\cos^2 l)\,X_1 = NP_x \qquad \Sigma(1-\cos^2 b\sin^2 l)\,Y_1 = NP_y \qquad \Sigma\cos^2 b\,.\,Z_1 = NP_z.$$

$$(5b)\quad \Sigma\cos^2 b\cos^2 l\,.\,X_1 = NQ_x \qquad \Sigma\cos^2 b\sin^2 l\,.\,Y_2 = NQ_y \qquad \Sigma\sin^2 b\,.\,Z_2 = NQ_z.$$

$$(6b)\quad NX = N(P_x + Q_x). \qquad NY = N(P_y + Q_y) \qquad NZ = N(P_z + Q_z).$$

Eliminating now P_x, P_y, P_z, Q_x, Q_y and Q_x we get:

$$X = \frac{\Sigma(1-\cos^2 b\cos^2 l)}{N}X_1 + \frac{\Sigma\cos^2 b\cos^2 l}{N}X_2.$$

$$Y = \frac{\Sigma(1-\cos^2 b\sin^2 l)}{N}Y_1 + \frac{\Sigma\cos^2 b\sin^2 l}{N}Y_2.$$

$$Z = \frac{\Sigma\cos^2 b}{N}Z_1 + \frac{\Sigma\sin^2 b}{N}Z_2.$$

As a consequence of the supposition of a uniform distribution over the galactic longitudes, the quantities l will disappear from the coefficients, for in any narrow zone of constant galactic latitude, we will have:

$$\Sigma\cos^2 l = \Sigma\sin^2 l = \tfrac{1}{2}\text{ number of stars.}$$

Therefore the preceding equations take the form:

$$(9)\quad \begin{aligned} X &= \left(\tfrac{1}{2} + \frac{\Sigma\sin^2 b}{2N}\right)X_1 + \left(\tfrac{1}{2} - \frac{\Sigma\sin^2 b}{2N}\right)X_2 \\ Y &= \left(\tfrac{1}{2} + \frac{\Sigma\sin^2 b}{2N}\right)Y_1 + \left(\tfrac{1}{2} - \frac{\Sigma\sin^2 b}{2N}\right)Y_2. \\ Z &= \frac{\Sigma\cos^2 b}{N}Z_1 + \frac{\Sigma\sin^2 b}{N}Z_2. \end{aligned}$$

As we know the numbers of stars up to the magnitude m in the zones of different galactic latitude, the coefficients of X_1, Y_1, Z_1, resp. X_2, Y_2, Z_2 (*i. e.* the weights with which the galactic components from the groups of material M_1 and M_2 must be combined to get the exact galactic components) may be computed. To get the components ξ, η and ζ relative to the geometrical centre of all the stars down to the magnitude m, we have thus to transform the quantities ξ_1, η_1, ζ_1, ξ_2, η_2, ζ_2 into the corresponding galactic components X_1, Y_1, Z_1, X_2, Y_2 and Z_2. Then we have to combine these in the manner explained above. The quantities X, Y and Z to once obtained may be transformed back to ξ, η and ζ. It thus becomes clear that they may be derived free from any hypothesis about the distribution of the stellar velocities in space.

CHAPTER II.

THE APPLICATION OF THE METHOD OF BRAVAIS TO THE PRESENT DETERMINATION OF THE APEX.

The definitive formulae used.

In the present determination of the apex the example of BRAVAIS with respect to the masses and radial peculiar velocities has been followed. The latter have been simply neglected; the former have been taken all equal to unity. We adopt therefore as origin of coordinates a point which is at rest relatively to the geometrical centre of gravity. With the distances, we have dealt in the way indicated in Chapter I. About this question a few remarks will be made presently.

The formulae definitively used thus get the following form:

$$(4)\quad \begin{cases} \Sigma(1-\cos^2\delta\cos^2\alpha)\,\xi - \Sigma\cos^2\delta\sin\alpha\cos\alpha\,.\,\eta - \Sigma\sin\delta\cos\delta\cos\alpha\,.\,\zeta = \\ \qquad\qquad \Sigma\varrho\,(15\cos\delta\sin\alpha\,.\,\mu_\alpha + \sin\delta\cos\alpha\,.\,\mu_\delta). \\ \Sigma(1-\cos^2\delta\sin^2\alpha)\,\eta - \Sigma\cos^2\delta\sin\alpha\cos\alpha\,.\,\xi - \Sigma\sin\delta\cos\delta\sin\alpha\,.\,\zeta = \\ \qquad\qquad \Sigma\varrho\,(-15\cos\delta\cos\alpha\,.\,\mu_\alpha + \sin\delta\sin\alpha\,.\,\mu_\delta). \\ \Sigma\cos^2\delta\,.\,\zeta - \Sigma\sin\delta\cos\delta\cos\alpha\,.\,\xi - \Sigma\sin\delta\cos\delta\sin\alpha\,.\,\eta = \\ \qquad\qquad \Sigma\varrho\,(-\cos\delta\,.\,\mu_\delta). \end{cases}$$

These equations may be obtained from the normal-equations of AIRY by combining those corresponding to the right-ascensions and those corresponding to the declinations in a suitable manner, after having given to every star a weight equal to ϱ^2. Therefore the method of AIRY applied in this manner must lead to the same results as that of BRAVAIS.

The materials.

The stars used have been chosen in such a way that they are distributed fairly uniformly over the whole of the sky. They belong to the following groups:

1) The stars of BRADLEY from the north pole to declination $-31°$ taken from Publ. 9 of the Astronomical Laboratory of Groningen. I have only rejected those stars between $-20°$ and $-31°$ which are also contained in NEWCOMB's catalogue of fundamental stars. There thus remain 2551 BRADLEY-stars.

2) The fundamental stars of NEWCOMB [1]) between $-20°$ and $-40°$ of declination. In all 114 stars.

3) A number of 198 fainter stars of TAYLOR (nearly all magnitudes between 6 and 9), between $-20°$ and $-35°$ of decl., for which the proper motions have been derived by Prof. KAPTEYN from the catalogue of TAYLOR DOWNING (eq. 1835) [2]) and that of TUCKER (eq. 1900) [3]) (these proper motions have not as yet been published).

4) The fundamental stars of NEWCOMB between $-40°$ and $-52°$. In all 38 stars.

5) A number of 490 stars from the catalogue of astrographic standard-stars by SIR DAVID GILL (eq. 1900) [4]). The stars also contained in NEWCOMB'S fundamental catalogue were excluded. Of the rest only the more reliable ones have been retained.

6) For the part of the sphere between $-52°$ and the south pole 225 stars of the „Fundamental-Catalog für die Zonenbeobachtungen am Südhimmel und südlicher Polar-Catalog für die Epoche 1900 von A. AUWERS". [5])

The total number of stars is 3616.

The data required for the individual stars.

As may be seen from the formulae (4) it will be necessary to know for every star the following quantities:

α and δ, the right-ascension and declination for a determined epoch, for which, in agreement with Publ. of the Lab. of Gron. n°. 9, we have chosen 1875;

μ_α and μ_δ, the proper motions in right-ascension and declination;

ϱ, the distance. Its estimation depends on the following quantities:

m, the magnitude, reduced to the Potsdam scale;

μ, the total proper motion in arc of the great circle;

ψ, the angle of position of the total proper motion, taken in the sense, determined by the formulae on page 11 of Publ. 9;

λ, the angular distance between the star and an approximate position of the apex which has been assumed in accordance with Publ. 9 [6]);

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[1]) Catalogue of fundamental stars for the epochs 1875 and 1900 reduced to an absolute system by SIMON NEWCOMB.

[2]) General catalogue of principal fixed stars by TAYLOR, revised and edited by DOWNING, Edinburgh, 1901.

[3]) Catalogue of Southern Stars of PIAZZI for the ep. 1900. Publications of the Lick-Observatory, vol. VI.

[4]) GILL, A catalogue of 8560 astrographic standard-stars between decl. $-40°$ and $-52°$ for the eq. 1900, London 1906.

[5]) Astron. Nachr. n°. 3431--'32.

[6]) See page 8 of Gron. Publ. n°. 9.

χ, the angle: North pole-Star-Antapex.

Finally

b, the galactic latitude, because it is our intention to include in our discussion, a separate treatment of zones of different galactic latitudes.

For the stars of BRADLEY all the required data have been taken from the catalogue in Publ. 9 of the Astron. Lab. at Groningen. The comparison with the proper motions in Newcomb's catalogue shows small systematic differences. They were not applied; it will be easy to take them into account in deriving our definitive results.

For the other groups of stars the value of the galactic latitude b and the quantities λ and χ were obtained by the aid of tables with the argument α and δ. The coordinates adopted for the pole of the Milky Way are those of Gould:

$$\alpha_{1875} = 190^\circ\ 20' \qquad \delta_{1875} = +\ 27^\circ\ 21'.$$

The quantities μ and ψ were derived from the μ_α and μ_δ by means of another table with the double argument $15\mu_\alpha \cos\delta$ and μ_δ [1]). As for the proper motions μ_α and μ_δ themselves and for the magnitudes m we have to consider the several groups separately.

Stars of Newcomb between — 20° and — 40° of declination.

The proper motions of these stars have been simply taken from Newcomb's Catalogue of fundamental-stars. As the magnitudes in this catalogue have been taken, wherever possible, from the Harvard Photometry [2]), a correction of $+\ 0^m.17$ [3]) is necessary to reduce them to the scale of Potsdam. As the hundredths of magnitudes have been neglected, it has been rounded off to $+\ 0^m.2$.

Stars of Taylor.

The proper motions of the faint stars derived from TAYLOR—DOWNING and TUCKER 1900 have been reduced by Prof. KAPTEYN to the system of NEWCOMB's fundamental-catalogue in the following way. As these faint stars were not contained in Newcomb's catalogue, it was necessary to derive first the systematic corrections for the bright stars of the same zone by comparing the proper motions of these with those of the stars contained in Newcomb's catalogue. Corrections were thus obtained for each hour of R. A. separately for the zones — 20° to — 30° and — 30° to — 35°. These were applied as a *first* correction for the faint stars. The means of the proper motions in R. A. thus corrected were then formed for every hour. If there is no error depending on the magnitude we must expect

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[1]) This table is the same as that which has been used for these quantities in Gron. Publ. no. 9 (see page 11). The quantities $15\mu_\alpha \cos\delta$ were there simply denoted by μ_α.

[2]) See page 243 of Newcomb's catalogue.

[3]) The value of $+\ 0^m.17$ for the correction Potsdam—Harvard has been adopted here and below in agreement with that applied in Publ. 8 (page 5 below).

the mean of the 24 average values thus obtained to be very nearly zero. As however in reality the value $-\ 0^s.0022$ was obtained, it was assumed that the correction for the error in the proper motions in R. A. due to *magnitude* must be

$$+\ 0^s.0022$$

on the average. This value was applied as a second correction. The values of the proper motions, obtained after the application of these corrections, have been provisionally adopted for the determination of the apex.

The magnitudes have been taken by Prof. KAPTEYN from Harvard, vol. XLV, when given in this catalogue. If not observed at Harvard they have been taken from the Cordoba-Durchmusterung or Schönfeld's Südliche Durchmusterung. In this case corrections have been applied in order to reduce them to the scale of Harvard. To get them reduced to the scale of Potsdam I have added the correction $+\ 0^m.17$.

Stars of Newcomb between — 40° and — 52° of declination

The proper motions have been taken from Newcomb's catalogue; to the quantities $15\ \mu_\alpha \cos\delta$ however a correction of $+\ 0''.026$ has been applied, as explained below (see stars of Gill)

The magnitudes have been treated in the same way as those of the stars in the zone between — 20° and — 40°.

Stars of Gill

Only the proper motions depending on a sufficient number of authorities have been used. The positions according to the several authorities have been reduced by Sir DAVID GILL to Newcomb's systems by applying constant correction both to the right-ascensions and to the declinations. To a few of them there have furthermore been applied periodic corrections.

A little difficulty was experienced for the proper motions in rightascension. It soon became obvious that they have a pronounced preference for negative values over nearly all the hours of R. A. Among 800 stars with rather well determined proper motions, depending nearly all upon the catalogues of TAYLOR (1835), GOULD (1875), CAPE (1880) and CAPE (1900), only 175 had a *positive* μ_α, all the rest were *negative*.

In a paper, published in the Astron. Nachr. [1]), Sir DAVID GILL treats the question of an apparent rotation of the brighter fixed stars as a whole with respect to the fainter stars as a whole for this zone. The same question has been treated also in the introduction to the catalogue of astrographic standard stars.

In order to examine whether the interpretation of the discordances pointed out in these papers is confirmed by a direct comparison of the mean values of

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[1]) A. N. 3800.

μ_a obtained from the brighter and fainter stars of the materials under discussion, I have taken the 800 stars just mentioned. After having rejected three of them having exceptionally large proper motions, there remained 441 stars brighter than 7.0, 356 fainter than that magnitude. For each of the two groups I have derived the mean value of μ_a for each hour of R. A. After that, if for the two groups we take the mean of the 24 values $\overline{\mu_a}$, we find:

$$\text{for the brighter stars:} \quad \overline{\mu_a} = -0^s.0027$$

$$\text{for the fainter stars:} \quad \overline{\mu_a} = -0^s.0030$$

These values do not confirm the supposition of a considerable systematic rotation of the brighter relative to the fainter stars.

Treating in the same way the proper motions furnished by NEWCOMB'S fundamental-catalogue in this zone, only taking the limits of the zone somewhat wider, we found from 61 stars (excluding e Eridani with an exceptional large proper motion):

$$\overline{\mu_a} = -0^s.0024.$$

The number of stars of the last group is somewhat small for drawing any very reliable conclusion. Still it will be granted, that the three results, together, show no evidence of the existence of a rotation of the bright stars relative to the faint ones. They rather show *a general* preference of *both* the bright and faint stars for negative values of μ_a. This must perhaps be ascribed to a systematic error in the system of Newcomb for this zone.

In trying to elucidate this point I made a comparison with AUWERS. From 50 stars I get:

$$\Delta\mu_a\,(\text{Auw.} - \text{Newc}) = -0^s.0010.$$

If this difference were applied as a correction the proper motions would become still more strongly negative. On the other hand however the amount of the difference shows that the uncertainty about the mean value of $\overline{\mu_a}$, left by the two systems, is such, that at least a large part of the values found above may still be ascribed to systematic error. Taking into account the little *a priori* probability of a systematic rotation of all stars in this zone as a whole it seems that where we have to choose between the possibilities of a real systematic rotation and a systematic error, the latter must be the more probable.

For the determination of the apex, happily it is not of any considerable importance whether the anomaly is due to a property of the faint stars only or of the whole of bright and faint ones; nor whether the phenomenon is real or only the effect of systematic error. The effect may be eliminated from the results by taking care that opposite hour-groups get into the equations with the same weights. With this precaution it will be indifferent whether a correction is applied for the anomaly in question or not; with respect to the estimated distances however, which depend on the adopted values of the proper motions, it seemed preferable to apply a correction, by which the anomaly is removed.

For this reason we have applied to all values of $15\,\mu_a \cos\delta$ both to the stars of Gill and to those of Newcomb in the same zone, a correction

$$\Delta\,(15\,\mu_a \cos\delta) = +0''.026$$

For the rest the proper motions have been taken unaltered from the respective catalogues.

For the purpose of reducing the magnitudes of the stars of Gill to the scale of Potsdam, the magnitudes, as given in the catalogue, have been compared, as far as possible, with those given in Harvard [1]). In this way the following corrections for the reduction to Harvard, denoted by H — G have been derived for the different classes of magnitudes. They have been slightly smoothed by a curve. By further adding a correction of $+0^m.17$, we got the reductions to the Potsdam scale, denoted by P — G.

m	H — G	P — G
	m	m
4.0 to 4.4	+ 0.04	+ 0.21
4.4 to 4.9	— 0.28	— 0.11
5.0 to 5.4	— 0.40	— 0.23
5.5 to 5.9	— 0.44	— 0.27
6.0 to 6.4	— 0.46	— 0.29
6.5 to 6.9	— 0.47	— 0.30
7.0 to 7.4	— 0.41	— 0.24
7.5 to 7.9	— 0.27	— 0.10
8.0 to 8.4	— 0.14	+ 0.03
8.5 to 8.9	+ 0.01	+ 0.18
9.0 to 9.4	+ 0.14	+ 0.31
9.5 to 9.9	+ 0.21	+ 0.38

As far as the magnitudes for the stars used were given in the HARVARD catalogue, these have been taken, after having added a correction of $+0^m.17$. For the rest the correction P — G was applied to the magnitudes.

Stars of Auwers.

For these stars the proper motions as given in the catalogue of AUWERS have been corrected for precession, adopting that of NEWCOMB. For this purpose values of Δm and Δn have been used in accordance with those taken in Publ. 9 of the Astron. Lab. of Groningen (pag. 4) *i. e.*

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[1]) Annals of the astron. observatory of Harvard college, vol. XXXIV, which according to page 53 of this catalogue, has been joined as closely as possible to the Harvard Photometry (vol. XIV, Pickering I according to the Publ. of Potsdam).

$$\Delta m = + 0^s.00035$$
$$\Delta n = + 0^s.00035 = + 0''.0053.$$

No other correction has been applied to the proper motions in the Catalogue of AUWERS. From a comparison of the proper motions given by this catalogue and those of NEWCOMB's fundamental catalogue, I found:

$$\Delta\mu_\delta(\text{Newc.} - \text{Auw.}) = - 0''.001.$$

This value cannot exceed its uncertainty and it did not seem worth while to apply it. For the derivation of the periodic corrections there is hardly a sufficient number of stars common to AUWERS and NEWCOMB. Such corrections have therefore not been applied. A constant correction $\Delta\mu_\alpha$ has not been applied as it could not be of influence upon the coordinates of the apex derived from the proper motions of this region of the sky. Moreover in our *definitive* solution all proper motions have been reduced to the system of AUWERS, as will be explained in Chapter III, so that for the definitive results it is of no importance, whether reductions to NEWCOMB's system are or are not applied in this stage of our work.

The magnitudes of two variable stars, ns. 184 and 367 of AUWERS have been taken from CHANDLER's Catalogue [1]), adopting the values corresponding to the maximum of brightness. All the other magnitudes have been taken from HARVARD [2]). These were reduced to Potsdam by a correction of $+ 0^m.17$.

Catalogue containing the data for the individual stars.

The data for the different stars used for the determination of the apex, except for those of BRADLEY, have been given in **Table I** at the end of the present publication. For the BRADLEY-stars they have been taken immediately from Gron. Publ. 9. In the first column of Table I the numbers of the stars have been given. In order to prevent any mistake the quantities $15\,\mu_\alpha \cos\delta$, which have been called simply μ_α in Publ. 9, have been denoted here by μ'_α. The other notations, the same as those used above, are in perfect accordance with those of Publ. 9. The quantities μ have been given in three decimals, but only the first two figures of the numbers may be considered to be calculated exactly. As for the rest the table explains itself.

The distances.

The distances have been estimated for the different stars in the way shortly explained in Chapter I It was therefore necessary to derive a table with the arguments λ and $p = \chi - \psi$, giving the quantities $f = \frac{1}{q}$ [3]) required for the reduction of the total proper motions to quantities of which the amount will be independent in the mean of the angle $\chi - \psi$. Referring to the Notes for details, we may sum up the derivation as follows. Let $\overline{\mu}_{\lambda, p}$ represent the mean value of the total proper motions μ, belonging to a determinate value of sin λ and $p = \chi - \psi$ [1]); $\overline{\mu}$ the mean value of all proper motions. Then in the first place the quantities q, which are defined by the equation

$$(10) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad q = \frac{\overline{\mu}_{\lambda, p}}{\overline{\mu}}$$

have been determined empirically from the stars of BRADLEY of Publ. 9. For that purpose the stars have been distributed into 4 groups according to the value of sin λ, *viz.*:

a)	$1.00 > \sin\lambda > 0.80$	(1588 st.)
b)	$0.80 > \sin\lambda > 0.60$	(500 st.)
c)	$0\,60 > \sin\lambda > 0.30$	(427 st.)
d)	$0.30 > \sin\lambda > 0.00$	(110 st.)

Each of these groups of stars has been subdivided into 7 groups according to the values of p, *viz.*:

1)	p betw.	0°	and	$\pm$ 15°
2)	„	$\pm$ 15°	„	$\pm$ 45°
3)	„	$\pm$ 45°	„	$\pm$ 75°
4)	„	$\pm$ 75°	„	$\pm$ 105°
5)	„	$\pm$ 105°	„	$\pm$ 135°
6)	„	$\pm$ 135°	„	$\pm$ 165°
7)	„	$\pm$ 165°	„	$\pm$ 180°

In each of the so formed groups *a* 1), *a* 2), . . . *b* 1) . . . etc. the mean value of μ has been determined. The values thus obtained have been considered to represent the value of $\overline{\mu}_{\lambda,p}$ corresponding to values of sin λ and p valid for the middle of the respective groups. Dividing these by the value $\overline{\mu}$ (the mean of the proper motions of *all* the stars used) we get a set of empirically determined values of q, for the different values of sin λ and p. As however these quantities run rather irregularly, mainly as a consequence of a few very large proper motions. I have smoothed them in various ways. In the first place a small number of exceedingly large proper motions, distributed over different values of p (6 proper motions in the zone $0.80 > \sin\lambda > 0.60$, 2 in the zone $0.60 > \sin\lambda > 0.30$) have been excluded. In the second place each of the groups *a* 1), *a* 2) . . . *b* 1) . . . . etc. has been subdivided into three equally numerous groups according to the amount of the proper motions. Treating separately the groups of what we will call the large, the mean and the small proper motions and dividing the mean $\overline{\mu}_{\lambda,p}$ obtained from the different groups by the mean value $\overline{\mu}$ of all the large,

[1]) Astron Journ. 379.
[2]) Harv. XXXIV.
[3]) See page 6.

[1]) For the sake of brevity we will in future denote the angle of position, which in Table I has been called $\chi - \psi$ in accordance with Publ. 9, by p.

resp. mean or small proper motions, we must get theoretically the same results from the three groups. By taking finally for each of the groups $a\,1)$, $a\,2)$ . . . $b\,1)$ . . . etc. the mean values of the q furnished by the three groups (of large, mean and small proper motions) we have finally obtained a set of values from which the irregularities have disappeared to a great extent. The values of q, determined empirically in this way, have been called q_0. They have been given, together with the values from the large, mean and small proper motions separately, in the Notes belonging to this Chapter [1])

On the other hand the values of q may also be derived theoretically if we make some assumptions about the distribution of the proper motions. For this purpose the following assumptions have been made:

1. In every part of space that contains a sufficient number of stars the peculiar proper motions are distributed equally over all directions and the amount of this proper motion is independent from the direction. This comes to adopting the hypothesis of the random distribution.

2. The amounts of the projections of the peculiar velocities *on a line* are arranged according to the law of accidental errors.

3. The mean values of the peculiar velocities are the same in different parts of the space.

If we call the velocity of the sun h, the mean value of the projection of the peculiar proper motions on a plane $\overline{n}$, it follows from our supposition that $\frac{h}{\overline{n}}$ is constant for every part of space.

Of course our first supposition is liable to objection. We have to consider, however, that the values of q are derived merely for the purpose of eliminating the effect of the sun's motion on the observed distribution of the proper motions. We thus have only to take into account such disturbances of the radial symmetry of this distribution as can be caused by the sun's motion. For the present purpose, therefore, I think that any error in our first assumption cannot have any serious effect.

As for the rest, if the assumptions made are not quite in accordance with the reality, they will only make the taxation of the distances for the individual stars slightly less exact than they would be if the peculiar proper motions could be taken into account instead of the reduced observed proper motions.

Now let:

Q = the mean linear value of the observed proper motions, corresponding to a determinate value of λ and ϕ,

h = the sun's linear velocity (as before):

$z = h \sin\lambda \cos\phi$;

m = measure of precision [2]) of the projections of the velocities upon a line, considered as accidental errors and let us take as the unit of velocity the quantity $\overline{n}$ = the mean value of the projections of the linear peculiar velocities on a plane.

We get the following equation:

$$(11) \quad . \quad Q = \frac{m(1 + 2m^2z^2) + mze^{-m^2\kappa^2} + (1 + 2m^2z^2)\int_0^{m\kappa} e^{-t^2}\,dt}{2m^2z + me^{-m^2\kappa^2} + 2m^2z\int_0^{m\kappa} e^{-t^2}\,dt}.$$

This formula may be derived from the formulae (26) and (27) in Publ. 5 of the Astr. Lab. of Gron. by taking $F(n)$ in such a way that it is in accordance with the distribution of the projections of the peculiar velocities supposed under 2. A simpler derivation has been given in the Notes [1]).

As a consequence of the assumptions made, the formula for Q is found to be quite independent of the distances of the stars. It may therefore be considered to be the same for stars at all distances. As further we may for stars at a determinate distance replace the ratio of the *linear* values corresponding to observed and peculiar proper motions by the *angular* values, we may conceive Q as the ratio of the mean angular values $\overline{\mu_{\lambda,\phi}}$, as defined above, and the mean angular value of the peculiar proper motions $\overline{\sigma}$ — which ratio will therefore be equal for groups of stars at all distances. Taking together the stars of all distances, we thus get:

$$(12) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad Q = \frac{\overline{\mu_{\lambda,\phi}}}{\overline{\sigma}},$$

$$(13) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad q = \frac{\overline{\sigma}}{\overline{\mu}}\,Q.$$

The values of Q may be derived for every value of λ and ϕ by means of the formula (11), if we adopt an approximate value of h. In this way the values of Q have been derived for the same values of $\sin\lambda$ and ϕ for which the values of q_0 have been determined empirically [2]). For m we adopted the value:

$$m = 0.886\ [3]).$$

In order to obtain the quantities q we have to multiply the quantities Q by the ratio $\frac{\overline{\sigma}}{\overline{\mu}}$ (see formula 13), which we will call C, therefore

$$(14) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad q = CQ.$$

The value of C cannot well be determined wholly theoretically as it depends partially on the distribution of the stars over the sky. For if all the stars were concentrated in a region near the apex or antapex, C would be nearly equal to unity; if on the other hand they were all distributed over the parallactic equator [4]) it would be considerably less. For this reason the quantity C has been deter-

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[1]) Notes to Chapter II 1).

[2]) Chauvenet, Spherical and practical astronomy II, page 485.

[1]) Notes to Chapter II 2).

[2]) Notes to Chapter II 3).

[3]) Notes to Chapter II 2).

[4]) This name is given by BRAVAIS to the great circle of which the apex is the pole.

mined empirically in such a way that the mean value $\overline{\mu'}$ of the reduced proper motions is equal to the mean value $\overline{\mu}$ of the observed proper motions [1]).

By means of formula (14) we get values of q which we will denote by q_c. If now the adopted value of k is in accordance with the best one, which can be derived from the materials used for the determination of the values of q_o, then the variation of the calculated values of q_c with the angles p must agree with that of the observed quantities q_o. As this was not the case, the originally adopted value of k has been improved by means of the empirical quantities q_o. After some trials [2]), I found that the value

(15) $$k = 0.750$$

produced a satisfactory accordance. Therefore this value has been finally adopted. By means of this value a set of quantities Q has finally been calculated for intervals of 10° for λ and p. Adopting further for C the value

$$C = 0.884$$

which has again been derived in the way quoted above, the corresponding values of q and the reciprocals

(16) $$f = \frac{1}{q} = \frac{1}{CQ}$$

could be calculated. In this way the **Table II** at the end of our paper was obtained.

In order to find the distances the observed total proper motions were now reduced to the corresponding quantities μ' by means of the formula

(17) $$\mu' = f\mu$$

in which the quantities f have been taken from **Table II** without interpolation. These quantities μ' may now be considered to be independent of the angle of of position p.

On the other hand a table has been formed for the derivation of the distances by extending the table on page 31 of Publ. 8 which gives the parallaxes of the stars with the arguments m and μ. Instead of the parallaxes however, the table was made to give the distances (unit of distance for $\pi = 0''.1$). From these tables the values of the distances have been obtained by taking μ' as argument instead of μ. The values thus obtained for the distances ϱ have been substituted into the formulae (4).

Table for the coefficients.

The coefficients of ξ, η and ζ in the first and those of $\varrho \,.\, 15\,\mu_a \cos\delta$ and $\varrho \,.\, \mu_\delta$ in the second members of the equations (4) have been taken from tables which give these quantities for intervals of 10^m in α and 1° in δ. In **Table III** at

[1]) Notes to Chapter II 3).
[2]) Notes to Chapter II 3).

the end of this publication they have been reproduced excepting $\cos^2\delta$, $\sin\alpha$, $\cos\alpha$ and $\cos\delta$.

Table giving the equations of Bravais for the individual groups.

In order to be able to derive the position of the apex from stars of different galactic latitudes separately, each of the groups mentioned on page 17 provisionally excluding BRADLEY's stars, has been subdivided into five groups defined by the following limits of galactic latitude:

(K)

I	Gal. lat.	$-90°$ to $-40°$
II	„	$-40°$ to $-20°$
III	„	$-20°$ to $+20°$
IV	„	$+20°$ to $+40°$
V	„	$+40°$ to $+90°$

For the stars of BRADLEY three declination-groups have first been formed, *viz.*:

(L)

Decl.	$+90°$ to $+52°$
„	$+52°$ to $-20°$
„	south of $-20°$

By this arrangement it became possible to combine the stars of BRADLEY between $+90°$ and $+52°$ with those of AUWERS between $-52°$ and $-90°$ and those south of $-20°$ with the stars of NEWCOMB and TAYLOR. The three groups (L) of BRADLEY have been also subdivided afterwards into the groups (K).

The data necessary for immediately finding the equations of BRAVAIS (4) for the different groups thus obtained have been given in **Table IV** at the end of this paper. The following notations have been used:

(18)
$$A = \Sigma\,(1 - \cos^2\delta\cos^2\alpha).$$
$$B = \Sigma\,(1 - \cos^2\delta\sin^2\alpha).$$
$$C = \Sigma\cos^2\delta.$$
$$D = \Sigma\cos^2\delta\sin\alpha\cos\alpha.$$
$$E = \Sigma\sin\delta\cos\delta\cos\alpha.$$
$$F = \Sigma\sin\delta\cos\delta\sin\alpha.$$
$$P = \Sigma\,\varrho\,(\sin\alpha \,.\, 15\,\mu_a\cos\delta + \sin\delta\cos\alpha \,.\, \mu_\delta).$$
$$Q = \Sigma\,\varrho\,(-\cos\alpha \,.\, 15\,\mu_a\cos\delta + \sin\delta\sin\alpha \,.\, \mu_\delta).$$
$$R = \Sigma\,\varrho\,(-\cos\delta \,.\, \mu_\delta).$$

In calculating P, Q and R, for the quantities $15\,\mu_a\cos\delta$ and μ_δ the centennial proper motions have been taken, expressed in seconds of arc.

The quantities A, B, C, D, E, F, P, Q and R have been given for each of the groups, obtained by subdividing the groups (L) of the stars of BRADLEY

and further the groups 2), 3), 4), 5) and 6) of page 17 into the groups (K). They have also been given further for the groups (L) of the stars of BRADLEY and the groups 2), 3), 4), 5) and 6) of page 17 as a whole (See **Table IV**: all stars together). Moreover for the stars of NEWCOMB in the zone between $-40°$ and $-52°$ and those of GILL, a separate solution has been made. In this separate solution NEWCOMB's stars have been given double the weight of GILL's stars. The weights of the individual stars were then taken in such a way that, in the final equations, opposite three-hour-groups entered with equal weights. This was done for the purpose of eliminating any constant error in μ_a or any magnitude-error remaining in the proper motions (see page 19 *etc.*).

CHAPTER III.

DISCUSSION OF THE RESULTS FROM ZONES OF DIFFERENT DECLINATION.

The components ξ, η and ζ of the solar motion and the coordinates A and D of the apex have been determined separately for each of the groups (L) of the stars of BRADLEY (see pag. 27) and further for the groups 2), 3), 4), 5) and 6) of page 17. This course was pursued with the double aim:

a. of more easily detecting remaining systematic errors;

b. of detecting any real differences that may exist between the apex furnished by the stars of different zones

Before doing so, however. a solution has first been made from all stars of BRADLEY together. They represent somewhat more than 70 percent of the whole of the materials used. The interest of this solution lies in the fact that it furnishes a comparison of the coordinates of the apex obtained by means of the method of BRAVAIS with those derived by Prof. KAPTEYN in A. N. N°. 3722 *from the same stars* by means of *other* methods. Our solution led to the values:

$$A = 27c°.0 \qquad D = +30°.8$$

whereas Prof. KAPTEYN obtained the following results:

by the meth. of ARGELANDER:	$A = 274°.3 \pm 1°.3$	$D = +27°.2 \pm 1°.4$
" " " " KAPTEYN :	$A = 267°.3 \pm 2°.2$	$D = +29°.4 \pm 1°.7$
" " " " AIRY :	$A = 274°.7 \pm 1°.7$	$D = +28°.5 \pm 1°.2$
mean value :	$A = 272°.2$	$D = +28°.4$.

The differences between the values obtained by the different methods are of the same order as that between the mean value and the value derived by the aid of BRAVAIS' method.

This fact might seem very surprising, seeing that all these older determinations professedly rest on the assumption of the hypothesis of the random-distribution of the proper motions — a hypothesis which as we said before is not nearly satisfied in nature. Still it is not difficult to show reasons which might have led us to expect that the difference of the result of the application of BRAVAIS' method from that of the older methods would be rather small.

For as we pointed out before, the equations which embody BRAVAIS' theory — in the case that we take the distances of all the stars to be equal — are the same as we get in AIRY's theory, under the same very common supposition, if we combine the equations of AIRY furnished by the right-ascensions and declinations in a suitable way Therefore, if we had supposed all distances equal, we would have been led to results nearly accordant with those obtained formerly by AIRY's method. Now it is true that we have not supposed the distances equal. But as this comes in the main nearly to introducing a different system of weights, very great changes are not to be expected. Furthermore Prof. KAPTEYN has shown[1]) that, owing to the circumstance that by the very common usage of „abridged calculation" the methods of AIRY, KAPTEYN, ARGELANDER will lead to results which can be but little different.

In **Table IV** the data necessary for the formation of the equations of BRAVAIS (4) for the separate groups have been collected (see page 27 and 28; the meaning of the notation „Combination Newcomb-Gill" has been also explained there).

By solving the equations the following values for the components of the solar velocity ξ, η and ζ and the coordinates of the apex A and D have been obtained.

TABLE 1.

		ξ	η	ζ	A	D	Number of stars.
Bradley	+ 90° to + 52°	— 0.2	— 19.1	+ 13.6	269°.4	+ 35°.5	297
„	+ 52° to — 20°	— 0.1	— 26.6	+ 15 3	269°.8	+ 29°.9	2115
„	south of — 20°	+ 1.7	— 27 7	+ 11 4	273°.5	+ 22°.3	139
Newcomb	— 20° to — 40°	— 0.2	— 27.1	+ 21.7	269°.6	+ 38°.7	114
Taylor	— 20° to — 35°	— 2.0	— 15.8	+ 20.2	262°.8	+ 51°.8	198
Newcomb	— 40° to — 52°	— 1.6	— 27.9	+ 20.8	266°.7	+ 36°.6	38
Gill	— 40° to — 52°	— 0.5	— 28.0	+ 27.9	269°.0	+ 44°.9	490
Comb. Newc.-Gill		— 0.3	— 27.4	+ 26.5	269°.4	+ 44°.0	
Auwers	— 52° to — 90°	— 2.3	— 26.7	+ 22.9	265°.1	+ 40°.5	225

The unit in which the components ξ, η and ζ are expressed is equal to the linear velocity of a star with a parallax of 0″.1 and a centennial motion of 1″. We will now discuss the values of the coordinates A and D obtained for the different groups.

[1]) Koninklijke Academie van Wetenschappen te Amsterdam. Proceedings of the section of Sciences, Jan. 1900.

The right-ascensions.

First of all we will contract the values obtained from the different sources in the same zones of declination. In the zone of declination — 20° to — 40° the results yielded by the stars of BRADLEY, NEWCOMB and TAYLOR were combined with the respective weights 1, 1 and ½ per star. In the zone — 40° to — 52° the result from the „Combination Newcomb-Gill" will be adopted[1]). The following abbreviations for the different sources will be used:

Br. = Bradley.
N. = Newcomb.
Tay. = Taylor.
Gi. = Gill.
Auw. = Auwers.

If now we arrange the results according to the declination of the zones we get:

TABLE 2.

Limits of decl.	Source	A	Numb. of stars.	p	R
+ 90° to + 52°	Br	269°.4	297	2.7	+ 0.2
+ 52° to — 20°	Br	269°.8	2115	12.2	+ 0.6
— 20° to — 40°	N, Br. and Tay	268°.9	451	2.4	— 0.3
— 40° to — 52°	N and Gi	269°.4	528	4.0	+ 0.2
— 52° to — 90°	Auw.	265°.1	225	2.0	— 4.1
	All stars together	269°.2	3616	23.3	

The weights p have been taken proportional to $\Sigma(1 - \cos^2\delta \cos^2\alpha)$ by dividing this coefficient by 100; only for TAYLOR this coefficient was divided by 200. The last line contains the weighted mean.

The reason of the smaller weight attributed to TAYLOR lies not only in the larger accidental errors, but also in the fact that the average brightness, consequently also the average proper motion is much smaller than in the case of the other authorities and it is evident that the smaller the motions the greater will be the influence of determined errors on the coordinates A and D.

The deviations R of the individual zones from the mean are shown in the last column. They are very small and no regular change with the declination

[1]) In future if NEWCOMB and GILL have been taken together in this zone, always the „Combination" denoted above will be used.

is shown. By comparing the results obtained for the individual groups separately with the mean value we get the values of R given in Table 3.

TABLE 3.

Limits of decl.	Source	A	Numb. of stars	p	R
+ 90° to + 52°	Br	269°.4	297	2.7	+ 0.2
+ 52° to — 20°	Br	269°.8	2115	12.2	+ 0.6
— 20° to — 31°	Br	273°.5	139	0.9	+ 4.3
— 20° to — 40°	N	269°.6	114	0.8	+ 0.4
— 20° to — 35°	Tay	262°.8	198	0.6	— 6.4
— 40° to — 52°	N	266°.7	38	0.3	— 2.5
— 40° to — 52°	Gi	269°.0	490	3.7	— 0.2
— 52° to — 90°	Auw.	265°.1	225	2.0	— 4.1

It will be remarked that in the zone — 40° to — 52° the residuals both for NEWCOMB and for GILL are negative, whereas in the preceding table the combination of the two gives a positive value of R. The cause lies of course in the different distribution of the weights over the three-hour-groups. In the present table every star has got a weight 1, in the preceding solution opposite three-hour-groups got equal weights in this zone. No importance need be attached therefore to the series of negative values in the second half of the column of R. If we consider the residuals as wholly accidental we may derive a mean error from them. Denoting the mean error [1]) belonging to the unit of weight by ε_0, we get:

(19) . . . . . $\varepsilon_0 = \pm 3°.4$

and as a preliminary result for A and its *mean* error, from all groups together

(20) . . . . . $A = 269°.2 \pm 0°.7$.

This value has been derived from 3517 stars if a star of TAYLOR is counted for $\frac{1}{2}$. Therefore if we denote by ε_{100} the mean error in the result derived from a group of 100 stars distributed over the whole of the sky we get:

(21) . . . . . $\varepsilon_{100} = \pm 4°.2$.

Comparing this value with those for the mean errors in A, determined in A. N. n°. 3722 from different groups of stars and taking into account the numbers of stars from which they have been derived, the result must be considered satisfactory.

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[1]) In future we will always work with *mean errors*.

The declinations

As may be seen at once from Table 1 the agreement of the declinations of the apex is less satisfactory than that of the right-ascensions. We again contract the results obtained from the different groups of stars in the same zone of declination. In the zone — 20° to — 40° the weights of BRADLEY, NEWCOMB and TAYLOR were again taken to be respectively 1, 1 and $\frac{1}{2}$ per star. In the zone from — 40° tot — 52° the „Combination Newcomb-Gill" has again been used. We thus get the following summary:

TABLE 4.

Limits of decl.	Source	D	Numb. of stars.	p	R
+ 90° to + 52°	Br.	+ 35°.5	297	0.6	+ 2°.2
+ 52° to — 20°	Br.	+ 29°.9	2115	18.0	— 3°.4
— 20° to — 40°	N. Br. and Tay	+ 35°.9	451	2.8	+ 2°.6
— 40° to — 52°	N. and Gi.	+ 44° 0	528	2.6	+ 10°.7
— 52° to — 90°	Auw.	+ 40°.5	225	0.4	+ 7°.2
	All stars together	+ 33°.3	3616	24.4	

The weights p were taken proportional to $\Sigma \cos^2\delta$ (the factor for TAYLOR having been halved). The last line contains the weighted mean, the last column the deviations from the mean. There seems to be a slightly systematic increase of the declination of the apex derived from stars of decreasing declinations.

For the individual groups we get the following table:

TABLE 5.

Limits of decl.	Source.	D	Numb. of stars	p	R
+ 90° to + 52°	Br.	+ 35°.5	297	0.6	+ 2°.2
+ 52° to — 20°	Br.	+ 29°.9	2115	18.0	— 3°.4
— 20° to — 31°	Br.	+ 22°.3	139	1.1	— 11°.0
— 20° to — 40°	N.	+ 38°.7	114	0.9	+ 5°.4
— 20° to — 35°	Tay.	+ 51°.8	198	0.8	+ 18°.5
— 40° to — 52°	N.	+ 36°.6	38	0.2	+ 3°.3
— 40° to — 52°	Gi.	+ 44°.9	490	2.4	+ 11°.6
— 52° to — 90°	Auw.	+ 40°.5	225	0.4	+ 7°.2

The three groups of BRADLEY taken separately do not show a change of the declination in the sense of the preceding table, but rather a change in the opposite direction. This seems to show that there must be a systematic difference between the fundamental systems of the three first and those of the remaining groups. In order to get somewhat more certainty about the reality of such a systematic difference, we derived a *mean* error in the supposition that all residuals are accidental. We get, denoting the mean error belonging to the unit of weight by ε_0:

(22) . . . . . $\varepsilon_0 = \pm 11''.9$.

Therefore as a provisional result for D and its mean error, from all stars together we get:

(23) . . . . . $D = +33''.3 \pm 2''.4$.

If ε_{100} represents a mean error analogous to that introduced in the discussion of the right-ascensions we get:

(24) . . . . . $\varepsilon_{100} = \pm 14''.3$

The value $\pm 2''.4$ of the mean error in D is enormously in excess of the corresponding value $\pm 0''.6$ derived by multiplying that of the preliminary value of A (20) by cos D. We are thus led to conclude that it seems inadmissable to attribute the residuals of the several values of D simply to accidental errors. A comparison of the value (24) found from the declinations with the values for the mean errors in the declinations found in A. N. n°. 3722 taking the numbers of stars into account, seems to confirm this conclusion.

If we combine on the one hand the stars of BRADLEY, on the other hand all the remaining stars and if we call D_1 and D_2 the two values of D, we get:

Stars of BRADLEY: $D_1 = +30''.8$
Remaining stars: $D_2 = +43''.6$
Therefore: $D_2 - D_1 = +12''.8$

Considering this difference to be real we may derive another value for ε_0, starting from the supposition of two really different values D_1 and D_2 for the two groups. We get:

(25) . . . . . $\varepsilon_0 = \pm 5''.8$
(26) . . . . . $\varepsilon_{100} = \pm 7''.0$

and denoting by ε_{D_1} and ε_{D_2} the mean error of D_1 and D_2 and by $\varepsilon_{D_2-D_1}$ the mean error in the difference $D_2 - D_1$:

$$\varepsilon_{D_1} = \pm 1''.3$$
$$\varepsilon_{D_2} = \pm 2''.7$$
$$\varepsilon_{D_2-D_1} = \pm 3''.0.$$

The values (25) and (26) of ε_0 and ε_{100} are considerable smaller than the values (22) and (24). Meanwhile the difference $D_2 - D_1$ is more than 4 times larger than the mean error of this quantity. It may thus be considered probable that the difference between D_1 and D_2 is not accidental. We have to choose therefore between two possibilities:

1°. there still remain systematic errors in the proper motions, large enough to explain the difference;

2°. there is a real difference between the motion of the centre of gravity of the northern and that of the southern stars.

The probability of systematic errors in the proper motions of the southern stars.

In order to get a better insight into the influence of systematic errors let us write our equations (4) in the form:

$$A\xi - D\eta - E\zeta = P$$
$$B\eta - D\xi - F\zeta = Q$$
$$C\zeta - E\xi - F\eta = R$$

The coefficients D, E and F are relatively small for most of the groups. This will generally be the case if the distribution of the stars over the different hours of right-ascension is not too irregular. If therefore we neglect the terms with these coefficients we get:

$$\xi = \frac{P}{A} \qquad \eta = \frac{Q}{B} \qquad \zeta = \frac{R}{C}.$$

In these expressions we have:

$$P = \Sigma_Q \quad (\sin\alpha \,.\, 15\,\mu_\alpha \cos\delta + \sin\delta\cos\alpha \,.\, \mu_\delta).$$
$$Q = \Sigma_Q\,(-\cos\alpha \,.\, 15\,\mu_\alpha \cos\delta + \sin\delta\sin\alpha \,.\, \mu_\delta).$$
$$R = \Sigma_Q\,(-\cos\delta \,.\, \mu_\delta).$$

We thus see that systematic errors in μ_α will influence only the components ξ and η and *not* the component ζ. The same may be said of periodic errors in μ_δ. On the contrary an error in μ_δ, *constant* for a whole zone, will get into the quantity R multiplied by $-\cos\delta$, that is always with the same sign and (supposing equality of distance) of the same amount, at least if the zone is sufficiently narrow. It will therefore influence the component ζ, consequently the value of D, which is determined by the equation

$$\text{tg}\,D = \frac{\zeta}{\sqrt{(\xi^2 + \eta^2)}}.$$

On the other hand, if we compare the values of ξ, η and ζ of Table 1 obtained from the different groups of stars, we see that the values of ξ differ only little from each other. The same is generally the case with the values of η. There is only exception for the value of η from BRADLEY $+52°$ to $+90°$ and from TAYLOR. The value of ζ on the contrary becomes at once larger if we pass from the stars of BRADLEY to those of NEWCOMB in the zone between $-20°$ and $-40°$ (see Table 5).

If we denote the values of ξ, η and ζ furnished by the stars of BRADLEY and those obtained from the remaining stars respectively by ξ_1, η_1, ζ_1 and ξ_2, η_2, ζ_2, we have:

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$\xi_1 = 0.0$ $\eta_1 = -25.3$ $\zeta_1 = +15.1$
$\xi_2 = -1.0$ $\eta_2 = -26.6$ $\zeta_2 = +24.7$.

So we see that the difference between D_1 and D_2 depends on the difference in the values of ζ_1 and ζ_2. If therefore it must be ascribed to systematic errors, these must be errors of $\mu\delta$, dependent only on the *declination* and not on the right-ascension.

For the stars of BRADLEY large systematic errors of this kind do not seem very probable. For the corrections of the proper motions of BRADLEY-AUWERS were derived by Prof. KAPTEYN independently from a system of fundamental stars, starting from the supposition that the stars with larger and those with smaller proper motions must really give the same values for the coordinates of apex. They agree in generally fairly well with those obtained by comparing the system of BRADLEY-AUWERS with that of NEWCOMB. This may be seen from the data in Publ. 9, page 5. As will be seen afterwards these corrections also differ but little from those necessary to reduce the proper motions of BRADLEY-AUWERS to the system of AUWERS given in the „Astronomische Abhandlungen Nr. 7" [1]). Thus three systems which we may denote as the systems of KAPTEYN, NEWCOMB and AUWERS confirm each other in the main for the part of the sky covered by the catalogue of BRADLEY.

On the other hand the probability of systematic errors in declination is much larger *a priori* for the southern stars, used for the determination of the apex, because the fundamental proper motions are generally much less exactly determined for these stars. We will consider the different zones separately.

Zone $-20°$ *to* $-40°$.

In this zone we have made use of BRADLEY-stars (taken from Publ. 9 of Gron.), of fundamental stars of NEWCOMB and of TAYLOR-stars reduced to NEWCOMB's fundamental system. By a comparison of the proper motions of Publ. 9 and those of NEWCOMB's fundamental catalogue, we find for this zone:

(27) . . . . $\Delta\mu\delta$ (N — Br.) $= -0''.0022 \pm 0''.0018$ (51 st.).

If two stars south of $-30°$, which give relatively large differences are excluded, we get:

$$\Delta\mu\delta \text{ (N} - \text{Br.)} = -0''.0014 \pm 0''.0016 \text{ (49 st.)}.$$

The correction $\Delta\mu\delta = -0''.0022$ applied to the BRADLEY-stars leads to a correction of D of about $+3°.5$.

Applying the same correction $\Delta\mu\delta$ with the opposite sign to the proper motions of the stars of NEWCOMB, (of which the adopted distances are somewhat smaller) we get a correction of D of about $-2°.5$.

We thus see that the difference $\Delta\mu\delta$ found above can at most explain a relatively small part of the difference in question. As the correction (27)

[1]) Tafeln zur Reduction von Sterncatalogen auf das System des Fundamentalcatalogs des Berliner Jahrbuchs von A. AUWERS.

has been derived only from stars in the zone between $-20°$ and $-31°$ it might be considered possible that in the zone between $-31°$ and $-40°$ the NEWCOMB proper motions have a larger systematic error which might explain the difference. If this is the case we must expect to find different values of D from the zone between $-20°$ and $-30°$ and that between $-30°$ and $-40°$. In reality we find from the stars of NEWCOMB between $-20°$ and $-30°$

$$D = +38.8 \text{ (68 st.)}$$

and from those between $-30°$ and $-40°$

$$D = +38°.6 \text{ (46 st.)}.$$

Though the numbers of stars in the two zones are small, there seems to be no reason for assuming a systematic difference between the proper motions of the two zones.

We can hardly escape the conclusion that the difference of 16° between the results from the BRADLEY- and NEWCOMB stars cannot be explained by systematic error and that at least the greater part of it must be due to accidental error.

As for the difference between the values of D obtained from NEWCOMB and TAYLOR, it does not seem admissable to attribute it to a systematic difference between the proper motions derived from the two sources, because the proper motions of TAYLOR have been reduced to the system of NEWCOMB as rigourously as possible (see page 18). As the stars of TAYLOR are distributed rather irregularly over the different hours of rightascension an imperfect correction of the error dependent on the magnitude must influence the components in the plane of X-Y and therefore the declination of the apex. For this reason I made another solution in which the TAYLOR stars were divided in three-hour-groups. Equal weights were given to opposite groups. The result is:

(28) . . . . . $A = 258°.0$ $D = +51°.4$

The declination is not sensibly changed.

Another reason for the difference might be supposed to lie in the fact that, owing to a greater percentage of faint stars, the average proper motion of the TAYLOR-stars must be smaller than that of the NEWCOMB stars. A common systematic error in the $\mu\delta$ must therefore *differently* affect the results obtained from the two sources. We will presently discuss the value of this supposition.

Zone $-40°$ *to* $-52°$.

In this zone all the proper motions have been reduced to NEWCOMB's system. As a comparison with BRADLEY is not possible, we will compare the system of NEWCOMB with other fundamental systems in order to see to what extent a systematic error may still exist. By a direct comparison of the proper motions we get:

(29) AUW. — N $= +0''.0056$ (50 st.)
BOSS — N $= -0''.0022$ (45 st. betw. $-37°$ and $-55°$).

The stars of AUWERS have been taken from the catalogue in A. N. 3431, those of BOSS from that in A. J. 448. A small supplementary correction has been applied in deriving the correction (29) in order to take into account the difference in the precessions used by AUWERS and NEWCOMB The correction would of course have been *zero*, if the stars were distributed uniformly over the zone.

Now if we apply a correction of $+0''.005$ to the values of μ_δ of NEWCOMB and GILL, we get from the „Combination NEWCOMB-GILL"

$$D = +34°.0$$

instead of

$$D = +44°.0 \text{ (see Table 4).}$$

On the other hand the stars of BRADLEY give (see page 29):

(30) . . . . . $D = +30°.8$.

We thus see that an error of the same order as the correction required to reduce the proper motions from the system of NEWCOMB to that of AUWERS (and therefore of the same order as the uncertainty left by the fundamental-system chosen), is sufficient for explaining the difference between the declination of the apex derived from the stars of BRADLEY and that from the stars of NEWCOMB and GILL.

Zone $-52°$ *to* $-90°$.

We ought not to attach any serious importance to the deviation of eht declination of the apex yielded by this zone from that obtained fromt he rest of our materials. For first: a little constant error in the values of μ_δ has a relatively large influence upon the component of ζ_1, as may easily be seen from the equations. Secondly the uncertainty left by the fundamental-catalogues in the system of the proper motions in declination is also rather large. By a direct comparison of the systems of proper motions of NEWCOMB and AUWERS, made in the same way as above, we get:

(31) . . . . . N — AUW. = $-0''.0010$

A comparison of BOSS and AUWERS is obtained from A. J. 448. If the zones $-55°$, $-60°$, $-65°$, $-70°$, $-75°$, $-80°$ and $-85°$ are combined with the weights 7, 6, 5, 4, 3, 2, 1, we get:

BOSS—AUW. = $-0''.0050$.

These corrections if applied to the proper motions of AUWERS would still further increase the declination of the apex. The amount however of the mutual differences may give us an idea of the uncertainty left in this zone by the choice of the fundamental system.

Summing up what has been said about the three zones south of $-20°$, we find that as yet no conclusive evidence is obtained in regard to any *real* difference in the apex furnished by the southern and northern stars.

There is however another consideration by which we may try to get a clue to the true significance of the systematic differences shown in table 5. This consideration starts from the evident fact that systematic error must cause a much

smaller change in the position of the apex derived from stars with considerable proper motion than in that found from slowly moving stars.

For this reason we will divide the southern stars in two groups according to the amount μ' of their reduced proper motion (17). We will again treat the different zones separately.

Zone $-40°$ *to* $-52°$.

The number of stars in this zone is particularly large; nearly all of them are stars of GILL. The declination of the apex (see Table 5) obtained from these stars is higher than in any of the other groups. In classing these stars in two divisions according to the amount of their reduced proper motions, we have taken care that the distribution over the different hours of right-ascension has become fairly well the same for both groups. This compelled us to vary to a certain extent the limit of μ' separating the two divisions. Treating both divisions separately we find from the group of larger proper motions:

$$D = +39°.2 \text{ (196 st.)}$$

from that of the smaller ones

$$D = +50°.4 \text{ (294 st.)}.$$

The value furnished by the larger motions, on which the influence of systematic error is smallest, indeed leads to the more normal declination of the apex.

If we might assume that the stars of greater and those of smaller motion ought to yield *the same apex*, we might even derive the amount of the systematic error of the proper motions in declination. We find:

(32) . . . . . $\Delta\mu_\delta = +0''.0057$

in that assumption. Adopting this value both groups lead to the value

(33) . . . . . $D = +34°.2$

which already agrees fairly well with the value obtained from the BRADLEY-stars. At the same time the correction (32) found in this way agrees nearly exactly with the correction (29) which would be required to reduce the proper motions to the system of AUWERS.

If therefore the values of μ_δ of GILL are reduced to the system of AUWERS we get the following results:

1°. the larger and the smaller reduced proper motions lead to the same value of D;

2°. this value of D agrees with the value derived from the BRADLEY-stars much more closely than before.

There thus seems to be a certain *a posteriori* probability that, for this zone. AUWERS' system of declinations is to be preferred to that of NEWCOMB.

Zone $-20°$ *to* $-40°$.

In this zone the larger value of D furnished by the rather faint stars of TAYLOR of which the proper motions are in the mean considerably smaller than those of the brighter stars of NEWCOMB may be considered as an indication in the same direction. A moment ago we found from the stars of Newcomb (see Table 5)

$$D = +38°.7$$

from those of TAYLOR treated in such a way, that opposite three-hour-groups get equal weights (see (28) on page 37)

$$D = + 51°.4$$

The correction $\Delta\mu_\delta$ necessary to bring the two determinations to agreement is found to be

(34) . . . . . $\Delta\mu_\delta = + 0''.0087$

the common value of D being

(35) . . . . . D = + 25°.3.

On the other hand the correction needed by both NEWCOMB and TAYLOR in order to reduce them to AUWERS' system is

(36) . . . . . $\Delta\mu_\delta = + 0''.0038$ (88 st.)

which, though considerably smaller is still in the same sense. In deriving this correction from the values of μ_δ of the stars common to Auwers and Newcomb a small supplementary correction has been applied to eliminate the influence of the difference of precession. The correction (36) leads to

(37) . . . . . D = + 33°.2

for NEWCOMB'S stars and to

(38) . . . . . D = + 42°.1

for those of TAYLOR. The divergence from BRADLEY is thus greatly diminished. Lastly, in order to reduce the μ_δ of BRADLEY in this zone to AUWERS' system, we have to add the value (36) to the difference (27). The value

$$\Delta\mu_\delta = + 0''.0016$$

thus obtained leads to

(39) . . . . . D = + 19°.8.

The reduction to AUWERS is thus seen to reduce considerably the divergences from the result (30) found from the whole of the BRADLEY-stars. The mean of the three values (37), (38) and (39), combined with the respective weights 1, $\frac{1}{2}$ and 1 per star is D = + 30°.5, a value accidentally in nearly perfect agreement with (30)

Zone — 52° *to* — 90°.

For the sake of completeness the stars of AUWERS in the region between — 52° and — 90° have also been subdivided in two groups. From the larger proper motions we obtained

D = + 37°.0 (101 st.)

from the smaller ones

D = + 44°.6 (124 st.).

Owing to the high declinations of the stars, the determinations are rather weak and no great weight ought to be attached to their difference. Still as far as the evidence goes it points the same way as that obtained from the other zones. The correction needed to bring the two values into agreement is

(40) . . . . . $\Delta\mu_\delta = + 0''.0020$

leading to the common value

(41) . . . . . D = + 34°.9.

On the other hand, reducing the proper motions of AUWERS to the system of NEWCOMB by applying the correction (31), derived above, we get:

(42) . . . . . D = + 43°.0.

The reduction to NEWCOMB thus increases the disagreement.

Taking all our results together, we see that corrections for the different groups of proper motions in declination of the southern stars, derived from the supposition that the stars of greater and those of smaller proper motions must lead to the same position of the apex, are found to be always positive. At the same time we see that these corrections lead to a better agreement between the values of D obtained from the northern and southern groups These facts may be regarded together as an indication for the existence of negative systematic errors in the used values of μ_δ of the southern stars. At the same time these corrections are in the same sense as those necessary to reduce the values of μ_δ from the system of NEWCOMB to that of AUWERS.

The probability of a real divergence of the values of the declination of the apex.

If on the other hand we assume that the difference found between the values of D from the northern and southern groups of stars has a cosmical cause, it ought to be ascribed to systematic motions, directed in the main toward the south pole, of the southern stars relative to those in the northern hemisphere. As the divergence is considerable only for the stars of small proper motion we have to assume that practically only the more distant stars show such a drift. We will now examine the effect of such a drift upon the components of the solar motion in the plane of $\xi\,\eta$ as compared to that upon the ζ-component.

Suppose that in a determined zone there is a drift of the stars between the right-ascensions α_1 and $\alpha_2 = \alpha_1 + \varphi$ and let us assume that the stars are uniformly distributed over the zone, a condition fairly well satisfied by the greater part of our materials. Where it is not, the same effect is obtained by attributing equal weights to the opposite three-hour-groups [1]). Further while we keep the ζ-axis unchanged let for a moment the ξ and η axes be taken to pass through points with the right-ascensions $\frac{\alpha_1 + \alpha_2}{2}$ and $6^h + \frac{\alpha_1 + \alpha_2}{2}$, and let the components of the solar motion in this system be ξ', η', ζ. Finally let $\Delta\xi'$, $\Delta\eta'$, $\Delta\zeta$ represent the change of the components caused by the drift. Then if A, B, C represent the

[1]) For the stars of Taylor and those of the zone between — 40° and — 52° we have therefore to consider the solutions in which opposite three-hour groups have got equal weights.

coefficient of ξ in the first, resp. that of η in the second and that of ζ in the third equation of BRAVAIS, we will have [1]):

$$(43) \quad . \quad . \quad \Delta\eta' = 0. \qquad \frac{\Delta\xi'}{\Delta\zeta} = \frac{4\sin\frac{1}{2}\varphi}{\varphi}\,\frac{C}{A+B}\ \text{tg. }\delta.$$

We distinguish two cases:

1st *Case*: $\varphi < \pi$. In this case the quotient $\frac{\Delta\xi'}{\Delta\zeta}$ will lie between the limits:

$$(44) \quad \begin{cases} \dfrac{\Delta\xi'}{\Delta\zeta} = \dfrac{2C}{A+B}\ \text{tg }\delta \text{ for } \varphi = 0 \text{ and} \\ \dfrac{\Delta\xi'}{\Delta\zeta} = \dfrac{4}{\pi}\,\dfrac{C}{A+B}\ \text{tg }\delta \text{ for } \varphi = \pi. \end{cases}$$

Now consider in the first place the zone $-40°$ to $-52°$ which (see Table 5) gave a value for the declination of the apex differing most widely from that yielded by the BRADLEY-stars. For the whole of the BRADLEY stars we have

$$(p) \quad . \quad . \quad . \quad . \quad \xi = 00. \qquad \eta = -25.3. \qquad \zeta = +15.1.$$

for the „Combination Newcomb-Gill" in zone $-40°$ to $-52°$ (see Table 1)

$$(q) \quad . \quad . \quad . \quad . \quad \xi = -0.3 \qquad \eta = -27.4. \qquad \zeta = +26\,5.$$

We thus have for the proportions of the differences $\Delta\xi$, $\Delta\eta$, $\Delta\zeta$, disregarding the signs:

$$\frac{\Delta\xi}{\Delta\zeta} = 0\,03 \qquad \frac{\Delta\eta}{\Delta\zeta} = 0.18.$$

These differences cannot be explained by a stardrift, for according to (44) this would require that the value of $\frac{\Delta\xi'}{\Delta\zeta}$ were contained between the limits 0.64 and 0.40 and probably rather nearer to the first value, because that is the value for small values of φ. For, as has been shown above, the deviations are greatest for the smaller proper motions, consequently for the more distant stars. Now, the greater the distances, the less probable it seems that there will be a star-drift embracing anything approaching twelve hours of right-ascension.

In zone $-20°$ to $-40°$ the TAYLOR-stars, reduced to NEWCOMB's system (equal weights being given to opposite three-hour-groups) yielded the values:

$$(r) \quad . \quad . \quad . \quad . \quad . \quad \xi = -3.3 \qquad \eta = -15.5 \qquad \zeta = +19.8$$

Comparing these with the values (p) we find the difference of the η to be such that an explanation of the deviation of the value of D by star-drift is not excluded. But owing to the large accidental errors of TAYLOR and the only moderate number of stars (198) the result is little trustworthy. Moreover not only is it, as it were, neutralized by the value of D obtained from the BRADLEY-stars (139 st.) in

[1]) For the proof see Note to Chapter III.

this zone, which shows a deviation in the opposite direction, but the NEWCOMB-stars (114 st.) give:

$$\frac{\Delta\xi}{\Delta\zeta} = 0.03 \qquad \frac{\Delta\eta}{\Delta\zeta} = 0.24$$

which values are incompatible with a star-drift which would require $\frac{\Delta\xi'}{\Delta\zeta}$ to lie between the limits 0.71 and 0.45.

Finally as to the zone $-52°$ to $-90°$. No inference of any importance can be drawn from the determination of ζ which in this zone is particularly uncertain.

2nd *Case*: $\varphi > \pi$.

Evidently by a star-drift extending over a region of more than 12^h in right-ascension a disturbance of the components ζ may be caused whilst the components ξ and η are hardly influenced [1]). The existence of such a drift however, by which the differences of the D found above might be explained may be considered little probable *a priori*. For as will be presently shown the corrections derived by Prof. KAPTEYN for the proper motions of BRADLEY-AUWERS from the assumption that the stars with small proper motions must give the same position of the apex as those with larger motions, agree in general fairly well with those obtained by a comparison with the systems of NEWCOMB and of AUWERS. It must be considered probable therefore that in the *northern* sky the correct proper motions of the more *remote* stars will yield a position of the apex which is in accordance with that furnished by *all* the northern stars together. In the *southern* sky we found that principally the stars of *small* proper motion lead to a divergent value of the declination of the apex. Therefore if we wish to explain this divergence by star-drift, we have to assume that, whereas the *whole* of the stars of the *northern* and at least the *nearer* stars of the *southern* hemisphere show no relative systematic motion, such a motion exists for a group of *remote southern* stars extending over more than 12^h in right-ascension, that is filling an enormous space, which in part surrounds an unaffected portion of the system. Such an assumption seems very improbable *a priori*.

Summing up, we find that there is but little evidence of a *real* difference between the declination of the apex for northern and southern stars.

Between the supposition of star-drift or systematic error, as the cause of the divergences in Table 5, the choice is not difficult. As long as further evidence is wanting, we can hardly do otherwise than make the remaining *systematic errors* in the adopted proper motions responsible for them.

The proper motions reduced to Auwers' system.

The preceding discussions point to the conclusion that, for the proper motions in *declination* of the southern stars AUWERS' fundamental system is pre-

[1]) This may be seen from the formulae for $\Delta\xi$, $\Delta\eta$ and $\Delta\zeta$ derived in Note to Chapter III.

ferable to that of NEWCOMB. For the northern hemisphere we have no such direct evidence, but the difference of the systems north of the equator is small and consequently the choice not very material. For the sake of greater homogeneity we will finally adopt AUWERS' system of μ_δ for the whole of the sky.

For the μ_δ of AUWERS-BRADLEY the data necessary for such a reduction have been taken from the tables of AUWERS in the Astronomische Abhandlungen no. 7: „Tafeln zur Reduction von Sterncatalogen". The corrections were found from the relation:

$$(45) \quad \Delta\mu(\text{AUW.-BR.}) = \frac{\frac{1}{2}(\Delta\,\text{Greenw. } 1860 + \Delta\,\text{Greenw. } 1864) - \Delta\,\text{BRADLEY}}{108}$$

For the region between 60° and 90° the correction has been determined by combining the corrections at 60°, 65°, 70°, 75°, 80°, 85° with the respective weights 6, 5, 4, 3, 2, 1. From the corrections found in this way those already applied by Prof. KAPTEYN (see Publ. no. 9, page 5) have been subtracted. The remaining corrections are given in the following table under the head AUW.—KAPT. For the sake of comparison I have added the differences NEWC.—KAPT. derived from the data on page 5 of Publ. 9.

TABLE 6.

DECL.	AUW.—KAPT.	NEWC.—KAPT.
South. decl.	+ 0″.0004	+ 0″.0010
0° to + 20°	— 0″.0017	— 0″.0009
+ 20° to + 40°	— 0″.0025	— 0″.0040
+ 40° to + 60°	— 0″.0012	+ 0″.0001
+ 60° to + 90°	+ 0″.0055	+ 0″.0014

The differences AUW.—KAPT. are mostly small and of the same order as the differences NEWC.—KAPT. There is only an exception for the zone 60° to 90° in which, owing to the very limited number of stars, the reliability of the determination is smallest.

Combining the corrections for the different zones with weights proportional to the numbers of stars in each of them, we find as an average correction of the proper motions in declination in the reduction to AUWERS:

$$(46) \quad \ldots\ldots\ldots\ldots \quad \Delta\mu_\delta = -0''.0007$$

in the reduction to NEWCOMB:

$$(47) \quad \ldots\ldots\ldots\ldots \quad \Delta\mu_\delta = -0''.0008.$$

Now let us compare the values of D obtained from the different groups of stars after having reduced the μ_δ respectively to the systems of AUWERS and NEWCOMB. For each of the three groups of stars of BRADLEY the average corrections (46) and (47) have been applied in the present discussion. [1]) For the southern stars which have been reduced originally to NEWCOMB the corrections

[1]) Afterwards the proper motions of BRADLEY have been more *exactly* reduced to the system of AUWERS (see page 50).

(29) and (36) have been used for the reduction of the μ_δ to AUWERS. Summing up we get:

TABLE 7.

Limits of decl.	Source	$\Delta\mu_\delta$
— 20° to — 40°	N	+ 0″.0038
— 20° to — 35°	Tay	+ 0″.0038
— 40° to — 52°	N	+ 0″.0056
— 40° to — 52°	Gi	+ 0″.0056.

On the other hand for the reduction of the proper motions of the stars of AUWERS between — 52° and — 90° to NEWCOMB the correction (31) originally neglected (see page 22), has been applied for the reduction to NEWCOMB.

The following table shows the resulting values of D_1, D_A and D_N denoting the values respectively obtained if we adopt AUWERS' or NEWCOMB's system.

TABLE 8.

Lim. of decl.	Source.	D_A	D_N	Numb. of stars	p	R_A	R_N
+ 90° to + 52°	Br	+ 39°.1	+ 39°.6	297	0.6	+ 7°.3	+ 6°.0
+ 52° to — 20°	Br	+ 31°.3	+ 31°.6	2115	18.0	— 0°.5	— 2°.0
— 20° to — 31°	Br	+ 23°.9	+ 24°.1	139	1.1	— 7°.9	— 9°.5
— 20° to — 40°	N	+ 33°.8	+ 38°.7	114	0.9	+ 2°.0	+ 5°.1
— 20° to — 35°	Tay	+ 42°.6	+ 51°.8	198	0.4	+ 10°.8	+ 18°.2
— 40° to — 52°	N	+ 28°.9	+ 36°.6	38	0.2	— 2°.9	+ 3°.0
— 40° to — 52°	Gi	+ 33°.4	+ 44°.9	490	2.4	+ 1°.6	+ 11°.3
— 52° to — 90°	Auw.	+ 40°.5	+ 43°.0	225	0.4	+ 8°.7	+ 9°.4
	All stars together	+ 31°.8	+ 33°.6	3616	24.0		

The mean values of the last line were obtained by combining the separate values with weights obtained by dividing the coefficient $\Sigma\cos^2\delta$ [1]) by 100. For TAYLOR $\frac{1}{4}\Sigma\cos^3\delta$ [2]) has been taken instead of this coefficient. The two last colums show the residuals.

As was to be expected the agreement between the values of D_A has become much better than before the reduction to AUWERS; ε_0, ε_{100} and ε_D,

[1]) See equations (4).

[2]) As, by the preceding discussions our confidence in TAYLOR has been rather weakened I resolved to diminish here the weight still more than was done originally.

being the quantities defined above (page 32) and ε_D the mean error in D_A we get from the R_A

(48) . . . . . $\varepsilon_0 = \pm 5^\circ.2$

(49) . . . . . $\varepsilon_{100} = \pm 6^\circ.3$

(50) . . . . . $\varepsilon_D = \pm 1^\circ.1$

Taking into account the numbers of stars, this last value is about of the same order as the values for ε in the declinations derived in A. N. 3721—'22.

If for the purpose of a comparison we derive the analogous quantities from the residuals R_N, we get:

(51) . . . . . $\varepsilon_0 = \pm 10^\circ.0$

(52) . . . . . $\varepsilon_{100} = \pm 12^\circ.2$

(53) . . . . . $\varepsilon_D = \pm 2^\circ.0$

They are very considerably larger.

If now we combine the values from the different groups of stars in the same zone with weights equal to the quantities p given in Table 8, we get:

TABLE 9.

Limits of decl.	Source	D_A	D_N	Numb. of stars	p	R_A	R_N
+ 90° to + 52°	Br.	+ 39°.1	+ 39°.6	297	0.6	+ 7°.3	+ 6°.0
+ 52° to — 20°	Br.	+ 31°.3	+ 31°.6	2115	18.0	— 0°.5	— 2°.0
— 20° to — 40°	Br. N. Tay.	+ 30°.7	+ 34°.2	451	2.4	— 1°.1	+ 0°.6
— 40° to — 52°	N. Gi	+ 33°.1	+ 44°.3	528	2.6	+ 1°.3	+ 10°.7
— 52° to — 90°	Auw.	+ 40°.5	+ 43°.0	225	0.4	+ 8°.7	+ 9°.4
	All stars together	+ 31°.8	+ 33°.6	3616	24.0		

Excepting for the extreme zones, the uncertainty of which is necessarily very considerable, the agreement of the values of D for the different zones of declination both in the northern and in the southern hemisphere, obtained after the reduction to AUWERS leaves nothing to be desired. On the contrary the reduction to NEWCOMB has hardly improved the results of Table 4. In the absence of other data for judging the relative reliability of the two fundamental systems, this fact has turned the balance in favour of that of AUWERS. In deriving final results we have definitively adopted AUWERS' system for the proper motions in *declination*.

From the good agreement of the values of A derived from the different

sources (see Table 3) we may conclude that in the proper motions in *right-ascension* there are probably in general no systematic errors of such a kind or of such an amount as to have a considerable influence on the position of the apex. From this point of view there would not seem to be any reason for the application of corrections to the values of μ_a There seem however to be traces of a small periodic systematic error in the values of μ_a of the stars of BRADLEY. In Publ. 9 no *periodic* corrections have been applied to the proper motions except those for precession. Therefore any periodic systematic errors in the original proper motions of BRADLEY-AUWERS are still contained in the proper motions of BRADLEY as here used. Now from the data of AUWERS in „Astron. Abhandl. no. 7" a slight periodic correction to the values of μ_a of BRADLEY-AUWERS may be derived. A direct comparison of the values of μ_a of Publ. 9 for the stars contained in NEWCOMB'S fundamentalcatalogue with the corresponding values obtained from this catalogue gives a correction in the same sense. The sense of the correction agrees also with that derived by Prof. KAPTEYN for the proper motions of BRADLEY-AUWERS in A. N. 3859 — '60. It is for these reasons that in deriving our definitive results we have applied corrections as well in *right-ascension* as in declination for the reduction of the proper motions of BRADLEY to the system of AUWERS, derived from the data given in „Astron. Abhandl. no. 7".

A reduction of the *whole* of our proper motions to the system of AUWERS — which might seem desirable for the sake of homogeneity — would require further the derivation of corrections for the greater part of the proper motions in *right-ascension* of our southern stars. For the proper motions of AUWERS between — 52° and — 90° no corrections are of course required. There thus would remain to be found corrections to the values of μ_a of NEWCOMB, GILL and TAYLOR in the zones of decl. — 40° to — 52° and — 20° to — 40°. In deriving such corrections we meet with a difficulty. The corrections in μ_a which chiefly affect the position of the apex are the *periodic* corrections. For the determination of such corrections however a sufficient number of data is not available. On the other hand a *constant* correction to the μ_a can be better determined but is of little importance. For it is evident that equal corrections applied to the μ_a of stars differing 12^h in right-ascension will neutralize each other in the second members of the equations of BRAVAIS. Therefore a constant correction to the μ_a will have no effect on the position of the apex if the stars are distributed equally over the different hours of right-ascension. Now this last condition is either well fulfilled for the stars used in the two zones or the same effect has been artificially secured by the choice of the weights with which the stars enter in the equations of BRAVAIS. There is an exception only for the stars of TAYLOR, which are distributed very irregularly over the different hours; but these influence the position of the apex only to a very small degree, as a consequence of the small weight attributed to them. This would seem to be sufficient reason for not applying such a correction to the μ_a. — But even if constant corrections of the μ_a were not eliminated, there would be objections to their application.

For the stars of GILL the original values of μ_a, wich where based on

the system of NEWCOMB, showed a distinct preference for negative values, a property which seemed to be common to the brighter and the fainter stars. We have discussed this anomaly in Chapter II and we came to the conclusion that it is probably attributable to systematic error. For that reason we have applied to the values of μ_α of these stars a positive correction of such an amount that the anomaly disappears. At the same time we have seen that it would be still increased by the application of a correction necessary for the reduction to AUWERS. It must thus be considered preferable not to destroy the effect of the correction already applied by a reduction of the values of μ_α to AUWERS.

A similar irregularity was shown by the proper motions of the faint stars of TAYLOR, derived originally by Prof. KAPTEYN from the positions of TAYLOR and TUCKER after reduction to NEWCOMB. In the same region the values of μ_α of the stars of NEWCOMB, which are much brighter in the mean, do not show a preference for negative values. The anomaly of the proper motions thus seems to be attributable to an error of magnitude for which Prof. KAPTEYN has applied a positive correction of such an amount that the mean value of μ_α obtained by combining the different hour-groups with equal weights, became zero (See page 18 *etc*).

The process is equivalent to the supposition that the peculiar proper motions in right-ascension of TAYLOR-TUCKER as well as those of the NEWCOMB-stars in the same region do not show a preference for positive or negative values. This correction too would be destroyed if to the *whole* of the TAYLOR-stars we were to apply corrections obtained from a comparison with the proper motions of the stars of AUWERS in the same region, which as a rule are relatively bright stars.

To sum up: there are objections to the application of any further *constant* correction to the μ_α of the stars of NEWCOMB, TAYLOR and GILL. Happily such a constant correction is practically without any influence on the position of the apex. More important would be the derivation of *periodic* corrections for the μ_α. The number of our stars in however hardly sufficient for the derivation of trustworthy corrections of this kind.

In order to avoid any misconception, I will remark that my statement, that the proper motions have been reduced to the system of AUWERS, must not be taken to the letter. It has been my aim to reduce the *centennial variations* to AUWERS as sharply as possible. After that the best results for the *proper motions* would have been obtained, if from these centennial variations a new constant of precession had been derived by means of which the proper motions themselves would then have been found. The difference however of this constant *with that of* NEWCOMB must be very small. This may be concluded from the data of **Table V** (see the values of $\Delta\mu_\alpha$, depending on the declination) [1]). Add to this the fact

[1]) As will be seen from the explanation of Table V in the beginning of Chapter IV these values represent the corrections for the equinox required for the reduction of the proper motions of Publ. 9 to AUWERS. These proper motions have been reduced to the equinox of NEWCOMB by Prof. KAPTEYN. As a rule the corrections are small.

that the precession is of no influence at all in a determination of the solar motion, if the stars are distributed uniformly over the whole of the sky — a condition, that is fairly well satisfied for the stars, used for the present determination of the apex — and it will be seen that the influence of such a small correction to the constant of precession upon the position of the apex must be wholly vanishing in our case. I have felt justified in retaining NEWCOMB's precession for these reasons.

CHAPTER IV.

DEFINITIVE RESULTS.

Corrections of the second members of the equations for the reduction of the proper motions to the system of Auwers.

For the derivation of the definitive results the corrections ΔP, ΔQ and ΔR required on account of the reduction of the proper motions to the system of AUWERS have been applied to the second members P, Q and R of the equations. For that purpose our groups (K) of page 27 according to the galactic latitude have been subdivided into groups of such an extent in right-ascencion and declination that for each of the subdivisions the values of α and δ and that of the correction to be applied to the proper motions may be considered to be constant.

For the stars of BRADLEY the divisions of Table 6 (see page 44) according to the declination have been each subdivided into groups of one hour in right-ascension. For each of the groups obtained in this way corrections $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ for the reduction to AUWERS have been calculated by means of the tables of AUWERS published in the „Astron. Abhandl. n°. 7". In Chapter III we derived already the corrections $\Delta\mu_\delta$ (Auw.—Kapt.) depending on the declination; these corrections which have been given in Table 6 (page 44) have been used. Moreover corrections for the μ_δ, depending on the right-ascension as well as corrections for the μ_α depending on the declination and such depending on the rightascension have been calculated by means of the formula (45). From the corrections $\Delta\mu_\alpha$ depending on the declination the constant correction $+0^s.00100$, applied already by Prof. KAPTEYN (see Gron. Publ. n°. 9, page 4) has been subtracted. In this way the corrections given in **Table V** have been found. I found the total corrections $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ by adding, for every area, obtained by means of the double subdivision, the corrections $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ depending on the declination to those depending on the right-ascension.

For the stars of NEWCOMB, TAYLOR and GILL the constant corrections given in Table 7 (page 45) have been applied for the μ_δ. For the μ_α no corrections have been applied for the reasons explained in the last part of Chapter III. For the southern stars of AUWERS no corrections are needed (see Chapter II, page 21 and 22).

For the calculation of the corrections ΔP, ΔQ and ΔR it was not necessary to apply corrections to the *individual proper motions*. For, according to what has been said already, our areas are so small that we may assume the corrections to be constant for the whole of their extent. For the stars of BRADLEY the subdivisions made have already been indicated above. For the southern groups, except that of AUWERS, for which no corrections need to be applied, the limits of declination are so narrow that they have been only further subdivided into groups of one hour of right-ascension. According to the equations (4) the computation of the corrections in question has to be made by means of the formulae:

$$\begin{aligned} \Delta P &= (\sin\alpha \,.\, 15\,\Delta\mu_\alpha \cos\delta + \sin\delta\cos\alpha\,\Delta\mu_\delta)\,\Sigma\varrho \\ \Delta Q &= (-\cos\alpha \,.\, 15\,\Delta\mu_\alpha \cos\delta + \sin\delta\sin\alpha\,\Delta\mu_\delta)\,\Sigma\varrho \\ \Delta R &= (-\cos\delta\,\Delta\mu_\delta)\,\Sigma\varrho. \end{aligned} \tag{54}$$

From the corrections ΔP, ΔQ and ΔR thus obtained for our subdivisions we easil get the analogous corrections for the larger groups.

Summing up: the groups of different galactic latitude have been kept separate for the calculation of the quantities ΔP, ΔQ and ΔR. For the stars of BRADLEY the corrections $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ necessary to reduce the proper motions to the system of AUWERS given in the „Astron. Abhandl. Nr. 7" have been applied. For the NEWCOMB-stars and those of TAYLOR and GILL only the constant corrections $\Delta\mu_\delta$ given in Table 7 (page 45) have been applied. For the southern stars of AUWERS no corrections have been applied.

The corrections ΔP, ΔQ and ΔR obtained for our different groups have been given in **Table VI**.

Results from the groups of different galactic latitude.

After the application of the corrections ΔP, ΔQ and ΔR the values of ξ, η and ζ and those of A and D have been determined separately from the groups of different galactic latitude.

For the weight of a single proper motion I adopted the following values

BRADLEY	1
NEWCOMB	1
TAYLOR	$\frac{1}{4}$
GILL	$\frac{1}{2}$
AUWERS	$\frac{1}{2}$

which in my opinion must be roughly in accordance with their accidental and remaining systematic errors. The coefficients and second members of the equations finally obtained have been given in **Table VII.** The notations used have the same signification as those of **Table IV** (see page 27).

By solving the equations the following values for the components of the solar motion have been obtained:

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TABLE 10.

Gal. lat.	ξ	η	ζ
— 90° to — 40°	— 2.4	— 23 8	+ 14 8
— 40° to — 20°	— 1.8	— 27.3	+ 18.3
— 20° to + 20°	— 0.4	— 28.6	+ 15.6
+ 20° to + 40°	— 1.5	— 26.2	+ 17.4
+ 40° to + 90°	— 1.0	— 27 9	+ 16.6
All stars together	— 1.1	— 26.8	+ 16.2

From these we derive the coordinates of the apex given in the following table.

TABLE 11.

Gal. lat.	A	D	R_A	R_D	p_A	p_D
— 90° to — 40°	264°.3	+ 31°.7	— 3°.3	+ 0°.5	1.9	4.4
— 40° to — 20°	266°.2	+ 33°.8	— 1°.4	+ 2°.6	2.9	3.3
— 20° to + 20°	269°.2	+ 28°.6	+ 1°.6	— 2°.6	9 8	7.5
+ 20° to + 40°	266°.7	+ 33°.5	— 0°.9	+ 2°.3	3.3	3.2
+ 40° to + 90°	267°.9	+ 30°.7	+ 0°.3	— 0°.5	2.1	4.2
All stars together	267°.7	+ 31°.2			20.0	22.6

The values of A and D from all stars together have been obtained by adding the equations for the individual groups and by solving the resulting system of equations.

By subtracting the values of A and D obtained from all stars together from those given by the individual groups the residuals denoted respectively by R_A and R_D have been found. A decided trend cannot well be recognised in them. Supposing the differences to be purely accidental we may derive the mean errors from the residuals. For that purpose weights have been given to the individual values of A and D, which are respectively equal to a hundredth part of the coefficients A and C (defined on page 27) in the equations of BRAVAIS. Denoting the mean errors of the values of A and D from all stars together by ε_A and ε_D, we get:

(55) $\varepsilon_A = \pm 0°.8$ $\varepsilon_D = \pm 1°.0.$

These values are in accordance with those of the means errors derived in Chapter III by means of a comparison of the values of A and D obtained from the zones of different declination; those of D after the reduction of the values of $\mu\delta$ to the system of AUWERS (see (20) on page 32 and (50) on page 46). There thus seems to be no reason to ascribe any reality to the differences found between the results from zones of different galactic latitude.

Results from groups of different type of spectrum.

In order to see whether a difference between the coordinates of the apex from stars of different type of spectrum can be ascertained, the stars of BRADLEY of Type I and Type II have been treated separately. The data for the formation

of the equations have been given in **Table VIII**, the second members P, Q and R reduced of course to the system of AUWERS. By solving the equations the following values of ξ, η and ζ have been obtained:

TABLE 12.

	Spectr.	ξ	η	ζ
Bradl.	Type I	— 1.0	— 27.6	+ 14.4
Bradl.	Type II	— 2.1	— 27.1	+ 17.3

from which we get:

TABLE 13.

	Spectr.	A	D	p_A	p_D	ε_A	ε_D
Bradl.	Type I	268°.0	+ 27°.5	7.4	8.6	± 1°.3	± 1°.7
Bradl.	Type II	265°.5	+ 32°.4	6.3	8.1	± 1°.4	± 1°.8.

The difference of the values of A and D obtained from the two groups is small. The weights p_A and p_D are obtained in the same way as before. As the scales of the weights are practically in accordance with those of Chapter III, we may use the mean errors of the unit of weight there found (see (19) on page 32 for the A and (48) on page 46 for the D). We thus obtain the values of ε_A and ε_D given in Table 13. It thus appears that the differences between the values of A and D, particulary the latter are slightly larger than might be expected from the values of ε_A and ε_D. Still the reality of the difference is of course more than doubtful.

Taking all the stars of BRADLEY (the „Remaining Stars" of Publ. 9 included) together we get, the proper motions being reduced to AUWERS, the following results:

(56) $\xi = - 1.3$ $\eta = - 26.9$ $\zeta = + 16.1$

(57) $A = 267°.3$ $D = + 30°.9$

These results too differ only very little from those obtained on page 52 from all the stars together (see Tables 10 and 11).

The definitive results from all the stars together.

As may be seen from the discussion of the difficulty afforded by the unknown radial velocities, it will be best, theoretically, to arrange the choice of the weights in such a way, that zones lying symmetrically relatively to the equator get into the equations with equal weights (see page 7 *etc.*) The condition however loses much of its importance now that we find that, after the reduction to AUWERS no marked differences remain between the values of the coordinates of the apex furnished by the different zones of declination. I will take advantage of this circumstance in order to adjudge weights somewhat

more in accordance with the real reliability of the sources of our proper motions than would be those strictly required by our theoretical condition. I have finally adopted:

weight 2 for a proper motion of Newcomb
„ $\frac{1}{4}$ „ „ „ „ „ Taylor-Tucker
„ 1 „ „ „ „ „ any other source.

The equations furnished by the several groups have therefore been multiplied by these numbers. They have then been added and by solving the resulting equations the coordinates of the apex have been determined.

If we count a star of NEWCOMB for 2, one of TAYLOR for $\frac{1}{4}$ star, we get the following numbers of stars in the different zones:

TABLE 14.

Decl.	Source	Numb. of stars
+ 52° to + 90°	Br.	297
+ 40° „ + 52°	Br.	219
+ 20° „ + 40°	Br.	626
0° „ + 20°	Br.	677
0° „ — 20°	Br.	593
— 20° „ — 40°	N., Br., Tay,	417
— 40° „ — 52°	N, Gi.	566
— 52° „ — 90°	Auw.	225

The symmetry with respect to the equator is defective. The regions from — 20° to — 40° and from — 52° to — 90° have a considerably smaller number of stars than the corresponding northern ones. There is however a sort of compensation in the zone from — 40° to — 52°, which contains a much greater number of stars than the corresponding northern zone. Alltogether we get 1819 stars on the northern, 1801 on the southern hemisphere, a remarkable close agreement.

In the zone from — 40° to — 52° I have used the „Combination Newcomb – Gill" (see page 30) after having applied a correction

(58) $$\Delta R = -2503.$$

for the reduction of the μ_i to AUWERS. To the quantities P and Q no correction need to be applied as opposite three-hour-groups have got equal weights in this combination. For the other groups of stars the corrections ΔP, ΔQ and ΔR, obtained by adding those derived for the partial groups of different galactic latitude, have been applied. They have been given in **Table VI** (see: All stars together).

The coefficients and second members of the definitive equations have been given in **Table IX**.

By solving the equations we find:

(59) $$\xi = -1.1 \qquad \zeta = -26.8 \qquad \zeta = +16.4.$$

(60) $$A = 267°.7 \qquad D = +31°.4.$$

These results agree almost absolutely with those found in Table 10 and Table 11 from all the stars together using weights based *exclusively* on the estimated reliability of the different groups of proper motions. They also agree with the results (56) and (57) obtained from the stars of BRADLEY separately. We thus see that the values of the coordinates are practically independent of the choice of the weights of the southern stars.

For the mean errors of the results (60) we will adopt the values

(61) $$\varepsilon_A = \pm 0°.8 \qquad \varepsilon_D = \pm 1°.1$$

in accordance with those obtained formerly from zones of different declination and those from zones of different galactic latitude (see (20) on page 32, (50) on page 46 and (55) on page 52).

If h represents the velocity of the sun, then the values (59) substituted in the formula:

$$h = \sqrt{(\xi^2 + \eta^2 + \zeta^2)}$$

lead to the value

(62) $$h = 31.5 \pm 0.9\,\varepsilon.$$

The value of the mean error adopted here has been derived by comparing the values of h obtained from the zones of different galactic latitude. As our unit of velocity is the linear velocity at right angles to the line of sight of a star with a parallax of 0″.1 and a centennial proper motion of 1″, we get, expressed in kilometers per second for the value of the solar velocity and its mean error:

(63) $$h = 14.9 \text{ KM} \pm 0.5 \text{ KM}.$$

The value of h found in this way is dependent on the values, adopted for the distances. These distances in their turn are determined by the parallaxes of the table in Gron. Publ. 8. The parallaxes of this table are based for a large part on the mean parallaxes of stars of different magnitude found in Astr. Nachr. 3487 (see page 5 of Publ. 8), which have been determined from the parallactic motions, adopting for the solar velocity the value:

(64) $$h = 16.7 \text{ KM} \pm 1.1 \text{ KM}$$

which has been derived by Prof. KAPTEYN from the radial velocities of 51 stars determined in Potsdam. The difference between the results (63) and (64) must be explained by the fact that we used a great number of stars in addition to those used by Prof. KAPTEYN. As was to be expected however, the difference is not very considerable.

Afterwards CAMPBELL has derived the value:

(65) $$h = 19.9 \text{ KM} \pm 1.5 \text{ KM}$$

from a number of 280 radial velocities. This value must be preferred at all events

to the value (63) [1]. This value together with the definitively adopted position of the apex (60) would lead to the improved values:

(66) $\xi = -0.7$ KM $\eta = -16.9$ KM $\zeta = +10.3$ KM.

For the determination of the coordinates of the apex we have used the formulae (4), which have been derived from the rigorous equations by neglecting the radial velocities. As has been explained in Chapter I this comes to the introduction of the hypothesis that the radial peculiar velocities of all the stars, transferred to the geometrical centre of gravity neutralize each other (see page 7). The position of the apex (60) found above rests therefore upon this hypothesis.

As already pointed out (page 7 *etc.*) there is a certain *a priori* probability that for stars which, like our stars are fairly uniformly distributed over the sky, the hypothesis will be at least approximately satisfied.

We may consider the hypothesis to be confirmed *a posteriori* by the good agreement of the positions of the apex obtained from stars in different parts of the sky. The hypothesis might be verified more positively by the agreement of the coordinates of the apex found from the astronomical and those from the spectroscopic velocities, if a suitable number of the latter quantities were available. In the case that there would appear to be a difference we might get the position of the apex resp. the components of the solar motion free from the hypothesis by combining the values of ξ, η and ζ found from both groups of proper motions in the manner indicated in the last part of Chapter I.

From his group of 280 spectroscopic velocities CAMPBELL finds the following values for the coordinates of the apex [2]):

(67) . . . $A = 277°.5 \pm 4°.8$. $D = \pm 20°.0 \pm 5°.9$.

Combining these with his value (65) for the solar velocity found from the same stars, the following values are found for the components of the solar velocity:

(68) . . $\xi = +2.4$ K.M. $\eta = -18.5$ K.M. $\zeta = +6.8$ K.M.

Campbell's values (67) for the coordinates of the apex differ rather considerably from the values (60) found in the present paper. As however Campbell's results were derived from a small number of exclusively very bright stars, whereas our results depend on a considerable number of stars which are much fainter in the mean the two results cannot be considered as quite comparable. For the same reasons it does not seem advisable to combine the values (68) for the components of the solar motion, derived by Campbell with the values (66) found above. Moreover a combination of the two sets of values is liable to the objection that in deriving the values (66) Campbell's value (65) for h has been used and therefore the values (66) may not be considered to be determined

[1]) That the mean error of (64) ls less than that of (65) depends on the fact that in deriving (64) another principle has been used.

[2]) Astrophys. Journal, vol. XIII, page 80 etc.

purely astronomically. It will therefore be better to defer a combination of the results from astronomical and spectroscopic data to the time that a suitable number of spectroscopic velocities of fainter stars will be available and that a value of h will have been obtained from purely astronomical data.

Happily the position of the apex is not in the least influenced by the uncertainty in the *absolute* values of ξ, η and ζ, as it depends only on the proportions of these quantities and not of the value of h. Finally I will not adopt the values (66) for the components of the solar velocity but only the values (60) for the coordinates of the apex and together with these the value (15) derived for $\frac{h}{n}$ [1]) from the BRADLEY-stars by a comparison of the values q_o and q_c (see page 26). It is true, that this value for $\frac{h}{n}$ is derived in the supposition of a random-distribution of the proper motions. But as it has been obtained by comparing the quantities q_o and q_c, the former of which is *independent* of that hypothesis, and as the differences of the two for the several zones show no systematic divergences we conclude that the influence of the systematic deviations from the random-distribution must be very small.

Summary of the results.

The results of this Chapter may be summed up as follows:

a) There seems to be no reason to admit the existence of real differences between the values of the coordinates of the apex from groups of stars of different galactic latitude.

b) For stars of different type of spectrum the results are somewhat more divergent. Still there seems to be no ground for assuming that there is a real difference.

c) All the stars together lead to he following values for the coordinates of the apex and their mean errors. They will be considered as the definitive result of our investigation.

$$A = 267°.7 \pm 0°.8. \qquad D = +31°.4 \pm 1°.1.$$

d) Between the h and $\bar{n}$ we have the following relation:

$$h = 0.750\ \bar{n}.$$

This relation was derived exclusively from the BRADLEY-stars.

[1]) As on page 26 the mean value $\bar{n}$ of the projections of the peculiar velocities on a plane is taken as unit of velocity, the quantity $\frac{h}{n}$ has been denoted there by h.

NOTES TO CHAPTER I.

1) Let the rectangular coordinates of an arbitrary star relatively to an arbitrarily chosen system of coordinates be x, y and z, the number of stars of group the N. We will call *geometrical centre of gravity* the point of which the coordinates $\overline{x}$, $\overline{y}$ and $\overline{z}$ are defined by the following equations:

$$N\overline{x} = \Sigma x \qquad N\overline{y} = \Sigma y \qquad N\overline{z} = \Sigma z.$$

Denoting the components of the velocity of the geometrical centre of gravity by $\Delta\overline{x}$, $\Delta\overline{y}$ and $\Delta\overline{z}$, those of an arbitrary star by Δx, Δy and Δz, we have therefore:

$$N\Delta\overline{x} = \Sigma\Delta x \qquad N\Delta\overline{y} = \Sigma\Delta y \qquad N\Delta\overline{z} = \Sigma\Delta z.$$

If now we take as origin of coordinates the geometrical centre of gravity or a point rigidly connected with it, the quantities $\Delta\overline{x}$, $\Delta\overline{y}$ and $\Delta\overline{z}$ will be identically *zero*. Therefore we get the equations:

$$\Sigma\Delta x = 0 \qquad \Sigma\Delta y = 0 \qquad \Sigma\Delta z = 0.$$

denoted by (3) on page 5.

2) To find formulae for the derivation of the components ξ, η and ζ from the spectroscopic proper motions, quite analogous to the formulae (4) which determine them from the astronomical ones, we will again start from the fundamental equations (3) of page 5.

If we denote the components of the total velocities of the stars, relative to the *moving* sun by δx, δy and δz, these equations may be written:

$$\Sigma(\delta x + \xi) = 0 \qquad \Sigma(\delta y + \eta) = 0 \qquad \Sigma(\delta z + \zeta) = 0.$$

In working out the values for δx, δy and δz, we have now to neglect the *tangential* peculiar proper motions. We will take the observed *radial* velocity v positive in a direction from the sun. The projection of the parallactic motion on a plane perpendicular to the line of sight, will be decomposed in the direction of the rightascension and that of the declination. The corresponding components will be called p and q. We get the equations:

$$\Sigma(v_x + p_x + q_x + \xi) = 0$$
$$\Sigma(v_y + p_y + q_y + \eta) = 0$$
$$\Sigma(v_z + p_z + q_z + \zeta) = 0$$

in which v_x, p_x, q_x denote the projections of the quantities v, p, q on the axis of x, etc. We have further:

$$p = \xi \sin\alpha - \eta \cos\alpha$$
$$q = \xi \sin\delta \cos\alpha + \eta \sin\delta \sin\alpha - \zeta \cos\delta.$$

Taking into account that the cosines of the angles between the axes of coordinates and the positive directions of p and q are represented by the expressions

$$\begin{array}{llll} -\sin\alpha, & +\cos\alpha, & 0 & \text{for } p \\ -\sin\delta\cos\alpha, & -\sin\delta\sin\alpha, & \cos\delta & \text{for } q \end{array}$$

we easily get the equations:

$$\xi\Sigma\cos^2\delta\cos^2\alpha + \eta\Sigma\cos^2\delta\sin\alpha\cos\alpha + \zeta\Sigma\sin\delta\cos\delta\cos\alpha + \Sigma v\cos\delta\cos\alpha = 0$$
$$\xi\Sigma\cos^2\delta\sin\alpha\cos\alpha + \eta\Sigma\cos^2\delta\sin^2\alpha + \zeta\Sigma\sin\delta\cos\delta\sin\alpha + \Sigma v\cos\delta\sin\alpha = 0$$
$$\xi\Sigma\sin\delta\cos\delta\cos\alpha + \eta\Sigma\sin\delta\cos\delta\sin\alpha + \zeta\Sigma\sin^2\delta + \Sigma v\sin\delta = 0.$$

which are identical with the equations (5) of page 12.

These equations have the form of the normal-equations, obtained from a set of equations of condition of the form:

$$\xi\cos\delta\cos\alpha + \eta\cos\delta\sin\alpha + \zeta\sin\delta + v = 0$$

which are identical with those used by CAMPBELL [1]). Although these equations of condition are the expression of the assumption that in deriving the sun's motion we may treat the peculiar radial velocities as accidental errors, we thus see that the normal-equations derived from them are independent of this supposition.

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[1]) Astrophys. Journal, vol. XIII, page 83 (eq. 4).

NOTES TO CHAPTER II.

1) Denoting by q_1, q_2 and q_3 the values of q obtained from the *large*, *mean* and *small* proper motions, and by q_0 the mean of these three quantities, the following values were empirically found for the different parallactic zones and angles of position (the latter representing the mean of the two limits of p).

TABLE 15.

$1.00 > \sin\lambda > 0.80$ (mean value = 0.94).

p	q_1	q_2	q_3	q_0	n
°					
7	1.11	1.17	1.28	1.19	364
30	1.24	1.21	1.15	1.20	539
60	0.96	0.98	0.87	0.94	220
90	0.80	0.81	0.80	0.80	158
120	0.76	0.76	0.85	0.79	164
150	0.49	0.57	0.62	0.56	95
173	0.94	0.56	0.50	0.67	48

$0.80 > \sin\lambda > 0.60$ (mean value = 0.72).

p	q_1	q_2	q_3	q_0	n
°					
7	0.88	0.95	1.06	0.96	83
30	0.95	1.05	1.06	1.02	160
60	0.99	0.83	0.84	0.89	102
90	0.97	0.91	0.82	0.90	57
120	0.94	0.83	0.75	0.84	37
150	0.65	0.78	0.58	0.67	39
173	0.63	0.51	0.72	0.62	14

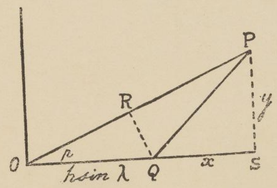
$0.60 > \sin\lambda > 0.30$ (mean value = 0.48).

p	q_1	q_2	q_3	q_0	n
°					
7	0.73	1.04	1.16	0.98	60
30	1.06	1.05	1.03	1.05	111
60	1.10	1.08	0.92	1.03	85
90	1.59	1.32	0.95	1.29	63
120	0.94	0.86	1.09	0.96	49
150	0.46	0.56	0.87	0.63	42
173	0.57	0.97	1.52	1.02	15

The numbers of stars used for the derivation of q_0 has been denoted by n. As a matter of fact the values of q_1, q_2, q_3 and q_0 have not been obtained immediately by taking the quotients $\frac{\overline{\mu\lambda,p}}{\mu}$, but by computing first the quotients $\frac{\overline{\mu\lambda,p}}{\mu\lambda}$ and afterwards the quotients $\frac{\overline{\mu\lambda}}{\mu}$, the latter quotients being calcuted by taking the large, mean and small proper motions together, without excluding any proper motion.

For the zone $0.30 > \sin\lambda > 0.00$ it was of no use to derive values for q_0. The influence of the small value of $h \sin\lambda$ for the stars in this region must be masked by the irregularities in the distribution of the proper motions.

2) To derive the formula for Q we imagine a group of stars, for which the values of $\sin\lambda$ lie between two limits, which are so narrow that $\sin\lambda$ may be considered as a constant. For all the proper motions we consider the linear values. Let v be the linear value corresponding to the observed proper motion, represented by the line O P in the figure. Let the peculiar proper motion Q P be decomposed in the direction of the antapex and at right angles to it and let these components be represented by the lines Q S $= x$ and P S $= y$ in the figure.



As a consequence of the supposition made under 2 on page 24, we may represent the probability that the projection of the peculiar proper motion on the axis of x will lie between x and $x + dx$ by the expression

$$\frac{m}{\sqrt{\pi}} e^{-m^2x^2}\, dx.$$

In the same way the corresponding probability for the y will be

$$\frac{m^2}{\sqrt{\pi}} e^{-m^2y^2}\, dy.$$

Therefore the probability that the projection on the axis of x will lie between x

and $x + dx$ and at the same time the projection on the axis of y between y and $y + dy$ will be

$$\frac{m^2}{\pi} e^{-m^2(x^2+y^2)}\, dxdy.$$

Now let us replace y by the angle of position p of the total proper motion as independent variable, using for that purpose the equation

$$y = (x + h \sin \lambda) \operatorname{tg} p.$$

By differentiating this, considering x as a constant and by then substituting the value thus obtained for dy in the expression found above, we get for the probability, that the x-projection will lie between x and $x + dx$ and at the same time the angle of position between p and $p + dp$ the expression:

$$(a) \qquad \frac{m^2}{\pi} \frac{(x + h \sin \lambda)}{\cos^2 p} e^{-m^2[x^2 + (x + h \sin \lambda)^2 \operatorname{tg}^2 p]}\, dxdp.$$

Now consider *all* the proper motions for which the angle of position lies between p and $p + dp$. The amount of the proper motion v will only depend on x, which we will suppose to vary between x_1 and x_2. Therefore as

$$v = (x + h \sin \lambda) \sec p$$

we will have as a consequence of (a) for the mean value Q of all proper motions, belonging to a determinate value of p the expression:

$$Q = \frac{\int_{x_1}^{x_2} \frac{(x + h \sin \lambda)^2}{\cos^2 p} e^{-m^2[x^2 + (x + h \sin \lambda)^2 \operatorname{tg}^2 p]}\, dxdp}{\int_{x_1}^{x_2} \frac{x + h \sin \lambda}{\cos^2 p} e^{-m^2[x^2 + (x + h \sin \lambda)^2 \operatorname{tg}^2 p]}\, dxdp}$$

or after some reductions:

$$Q = \frac{\int_{x_1}^{x_2} (x + h \sin \lambda)^2 e^{-m^2(x \sec p + h \sin \lambda \operatorname{tg} p \sin p)^2}\, dx}{\cos p \int_{x_1}^{x_2} (x + h \sin \lambda)\, e^{-m^2(x \sec p + h \sin \lambda \operatorname{tg} p \sin p)^2}\, dx}.$$

If now we put:

$$x \sec p + h \sin \lambda \operatorname{tg} p \sin p = z$$

we get, denoting the limits of z by z_1 and z_2:

$$Q = \frac{\int_{z_1}^{z_2} (z + h \sin \lambda \cos p)^2 e^{-m^2 z^2} z d}{\int_{z_1}^{z_2} (z + h \sin \lambda \cos p)\, e^{-m^2 z^2}\, dz}.$$

To find the values of z_1 and z_2, we have to consider that we have:

for $90° > p > -90°$	$x_1 = -h \sin \lambda$	$x_2 = +\infty$
therefore:	$z_1 = -h \sin \lambda \cos p$	$z_2 = +\infty$
for $270° > p > -90°$	$x_1 = -h \sin \lambda$	$x_2 = -\infty$
therefore:	$z_1 = -h \sin \lambda \cos p$	$z_2 = +\infty$.

The limits of z are therefore independent of p. Again putting in accordance with what was done in Chapter II

$$h \sin \lambda \cos p = \kappa$$

we get:

$$(b) \qquad Q = \frac{\int_{-\kappa}^{\infty} (z + \kappa)^2 e^{-m^2 z^2}\, dz}{\int_{-\kappa}^{\infty} (z + \kappa)\, e^{-m^2 z^2}\, dz}.$$

It is well known, that the integrals entering in this expression are easily reduced by partial integration to the integral $\int e^{-t^2} dt$. We get:

$$(c) \qquad Q = \frac{\frac{\sqrt{\pi}}{4m^3} + \frac{1}{2m^3} \int_0^{m\kappa} e^{-t^2} dt + \frac{\kappa}{2m^2} e^{-m^2\kappa^2} + \frac{\kappa^2}{m} \int_0^{m\kappa} e^{-t^2} dt + \frac{\kappa^2 \sqrt{\pi}}{2m}}{\frac{1}{2m^2} e^{-m^2\kappa^2} + \frac{\kappa}{m} \int_0^{m\kappa} e^{-t^2} dt + \frac{\kappa\sqrt{\pi}}{2m}}.$$

The value of m is obtained as follows:

Let $\bar{t}$ represent the average value of the linear velocities projected on a line, then the quantities $\bar{t}$ and m are evidently the same quantities which, in the theory of errors, are called:

η = mean of the errors all taken positively;
h = measure of precision.

Therefore the well known formula

$$h = \frac{1}{\eta\sqrt{\pi}}$$

proves that we must have

$$m = \frac{1}{\bar{t}\sqrt{\pi}}.$$

But as we took see page 24 *etc.* $\bar{n}$ as our unit, we have according to Gron. Publ. N°. 5, form. (8)

$$\bar{t} = \frac{2}{\pi}.$$

Therefore

$$m = \tfrac{1}{2}\sqrt{\pi} = 0.886.$$

By the aid of this value we may write the formula (c) in the form:

$$Q = \frac{m(1 + 2m^2\kappa^2) + m\kappa e^{-m^2\kappa^2} + (1 + 2m^2\kappa^2) \int_0^{m\kappa} e^{-t^2} dt}{2m^3\kappa + me^{-m^2\kappa^2} + 2m^2\kappa \int_0^{} e^{-t^2} dt}.$$

which served for the calculation of the quantities Q.

3) In order to derive the values of Q by means of the formula (11) we have provisionally adopted

$$h = 0.685$$

in accordance with the value for $\frac{\bar{n}}{h}$ derived by Prof. KAPTEYN for the stars of BRADLEY in A. N. 3487 [1]). With the aid of this value of h the values of κ have been calculated

[1]) As $\bar{n}$ has been taken as unit of velocity, the adopted value of h corresponds to the quantity $\frac{h}{n}$ of A. N. 3487.

for all the values of sin λ and p, for which the quantities q_0 have been calculated empirically. As we have $m = 0.886$ (see Note 2) all the quantities wanted for the calculation of the values of h are now known. For the calculation of the integrals, Table IX of CHAUVENET [1]) has been used. We thus obtained the following values of Q.

TABLE 16.

Table of Q ($h = 0.685$).

p \ sin λ	0.94	0.72	0.48
°			
7	1.32	1.24	1.15
30	1.27	1.20	1.13
60	1.15	1.12	1.07
90	1.00	1.00	1.00
120	0.87	0.90	0.93
150	0.79	0.84	0.88
173	0.77	0.81	0.87

In order to get the corresponding values of q we must now determine the quantity C (see formula (14) of page 25). As we have $C = \frac{\bar{\sigma}}{\bar{\mu}}$, the factor C might be calculated by means of the mean value $\bar{\sigma}$ of all the peculiar proper motions, which in thits turn might be obtained by multiplying the mean value of all the quantities τ, given in Gron. Publ. 9, by $\frac{\pi}{2}$ (see the formula 8 on page 14 of Gron. Publ. 5). For our purpose however the factor C is better determined in another way. For the factors f, if they have been well determined, will be such, that for the different values of λ and p the mean values of the reduced proper motions μ' agree as closely as possible with the mean value of all proper motions μ. If therefore we call the mean value of all the reduced proper motions $\bar{\mu}'$, of all observed proper motions $\bar{\mu}$, we have the condition

(d) . . . . . $\bar{\mu}' = \bar{\mu}$.

From this condition the value of C may be determined. If we call n the number of proper motions belonging to a determinate value of λ and p, the total number of all the proper motions N, then we have as a consequence of the definition of μ':

$$\bar{\mu}' = \frac{\sum_{\lambda,p} n f \bar{\mu}_{\lambda,p}}{N}.$$

[1]) Chauvenet, spherical and practical astronomy, vol. II.

Therefore the condition (d) becomes:

$$\frac{\sum_{\lambda,p} n f \mu_{\lambda,p}}{N} = \bar{\mu}.$$

According to formula (16) of page 26 this may be written:

$$\frac{1}{C}\,\frac{\sum_{\lambda,p} n \frac{1}{Q} \bar{\mu}_{\lambda,p}}{N} = \bar{\mu},$$

therefore:

(e) . . . . . $C = \frac{\sum_{\lambda,p} n \frac{1}{Q} \bar{\mu}_{\lambda,p}}{N\bar{\mu}}$.

In this expression we introduce for the $\bar{\mu}_{\lambda,p}$ and the $\bar{\mu}$ their values as they were empirically determined, for Q the values from Table 16; we then get

$$C = 0.892.$$

With this value formula (14) of page 25 and Table 16 lead at once to the following table for the q_e.

TABLE 17.

Table of $q_e = CQ$. ($h = 0.685$).

p \ sin λ	0.94	0.72	0.48
°			
7	1.18	1.11	1.03
30	1.13	1.07	1.01
60	1.03	1.00	0.95
90	0.89	0.89	0.89
120	0.78	0.80	0.83
150	0.70	0.75	0.79
173	0.69	0.72	0.78

In the following table these values are compared with those empirically determined.

TABLE 18.

$1.00 > \sin \lambda > 0.80$ (mean 0.94).

p	mean p	q_o	q_c	O−C	n
° °	°				
0 to 15	7	1.19	1.18	+ 0.01	364
15 to 45	30	1.20	1.13	+ 0.07	539
45 to 75	60	0.94	1.03	− 0.09	220
75 to 105	90	0.80	0.89	− 0.09	158
105 to 135	120	0.79	0.78	+ 0.01	164
135 to 165	150	0.56	0.70	− 0.14	95
165 to 180	173	0.67	0.69	− 0.02	48

$0.80 > \sin \lambda > 0.60$ (mean 0.72).

p	mean p	q_o	q_c	O−C	n
° °	°				
0 to 15	7	0 96	1.11	− 0.15	83
15 to 45	30	1.02	1.07	− 0.05	160
45 to 75	60	0.89	1.00	− 0.11	102
75 to 105	90	0.90	0.89	+ 0.01	57
105 to 135	120	0.84	0.80	+ 0 04	37
135 to 165	150	0.67	0.75	− 0.08	39
165 to 180	173	0.62	0.72	− 0.10	14

$0.60 > \sin \lambda > 0.30$ (mean 0.48).

p	mean p	q_o	q_c	O−C	n
° °	°				
0 to 15	7	0.98	1.03	− 0.05	60
15 to 45	30	1.05	1.01	+ 0.04	111
45 to 75	60	1.03	0.95	+ 0.08	85
75 to 105	90	1.29	0.89	+ 0.40	63
105 to 135	120	0.96	0.83	+ 0.13	49
135 to 165	150	0.63	0.79	− 0.16	42
165 to 180	173	1.02	0.78	+ 0.24	15

In the first place we will consider the variation of q_o and q_c with the value of $\sin \lambda$.

As may be seen from Table 18 the difference O−C is mostly negative in the second, positive in the third zone.

The cause must be sought in the *mean* value of $\frac{\bar{\mu\lambda}}{\mu}$ in the different zones of $\sin \lambda$.

For we have:

	$\frac{\bar{\mu\lambda}}{\mu}$	Number of stars.
$1.00 > \sin \lambda > 0.80$	1.02	1588
$0.80 > \sin \lambda > 0.60$	0.92	498
$0.60 > \sin \lambda > 0.30$	1.02	427

The consequence is that the empirically determined values of q_o must become relatively small in the second, relatively large in the third zone:

In deriving the values of q_o, firstly the quotients $\frac{\mu\lambda . p}{\mu\lambda}$ have been calculated for the individual zones, excluding a small number of stars with exceedingly large proper motions. In this way some of the larger irregularities in the values of $\frac{\bar{\mu\lambda , p}}{\mu\lambda}$ are removed. In order to reduce the quantities thus obtained to the mean value of all the proper motions which we have taken as unity, the values found for the different zones have been multiplied by the quantities $\frac{\bar{\mu\lambda}}{\mu}$. These last quantities have been derived from all stars of Publ. 9 without excluding any proper motion.

If now we exclude the three stars having largest proper motions and distributed in the following way:

	p	μ
$1.00 > \sin \lambda > 0.80$	0° to ± 15°	3″.76
$0.60 > \sin \lambda > 0.30$	± 15° to ± 45°	5″.21
$0.60 > \sin \lambda > 0.30$	± 75° to ± 105°	4″.05

we get:

	$\frac{\bar{\mu\lambda}}{\mu}$
$1.00 > \sin \lambda > 0.80$	1.05
$0.80 > \sin \lambda > 0.60$	0.97
$0.60 > \sin \lambda > 0.30$	0.85

We thus see that the way in which the values of $\frac{\bar{\mu\lambda}}{\mu}$ vary with $\sin \lambda$ has been considerably altered by the exclusion.

Multiplying the values of $\frac{\bar{\mu\lambda , p}}{\mu\lambda}$, obtained for the individual zones, by the *new* values of $\frac{\bar{\mu\lambda}}{\mu}$, we find the values of q_o given in the following table together with those of q_c.

TABLE 19.

$1.00 > \sin\lambda > 0.80$ (mean 0.94)

p	mean p	q_o	q_c	O – C	n
° °	°				
0 to 15	7	1.22	1.18	+ 0.04	364
15 to 45	30	1.23	1.13	+ 0.10	539
45 to 75	60	0.97	1.03	– 0.06	220
75 to 105	90	0.82	0.89	– 0.07	158
105 to 135	120	0.81	0.78	+ 0.03	164
135 to 165	150	0.58	0.70	– 0.12	95
165 to 180	173	0.68	0.69	– 0.01	48

$0.80 > \sin\lambda > 0.60$ (mean 0.72).

p	mean p	q_o	q_c	O – C	n
° °	°				
0 to 15	7	1.02	1.11	– 0.09	83
15 to 45	30	1.08	1.07	+ 0.01	160
45 to 75	60	0.94	1.00	– 0.06	102
75 to 105	90	0.95	0.89	+ 0.06	57
105 to 135	120	0.89	0.80	+ 0.09	37
135 to 165	150	0.71	0.75	– 0.04	39
165 to 180	173	0.66	0.72	– 0.06	14

$0.60 > \sin\lambda > 0.30$ (mean 0.48).

p	mean p	q_o	q_c	O – C	n
° °	°				
0 to 15	7	0.82	1.03	– 0.21	60
15 to 45	30	0.88	1.01	– 0.13	111
45 to 75	60	0.86	0.95	– 0.09	85
75 to 105	90	1.07	0.89	+ 0.18	63
105 to 135	120	0.80	0.83	– 0.03	49
135 to 165	150	0.53	0.79	– 0.26	42
165 to 180	173	0.85	0.78	+ 0.07	15

The differences O—C are now no longer systematically positive or negative in the first and second zone; in the third they have become negative, instead of positive. We thus find that the anomaly is wholly explained by the irregularities introduced by a few extremely large proper motions.

For this reason it seems preferable to adopt a system of values for q which vary with $\sin\lambda$ in the way postulated by theory. The procedure has the advantage of accommodating itself closely to the real variations in the average proper motion in the different parts of the sky, whereas, if we use the empirical values of q the average proper motion is artificially brought to equality over the whole of the sphere.

We will therefore adopt a system of values of q for which the variation with $\sin\lambda$ is determined by the theoretically calculated values. For that purpose we multiply the values of q_o for every zone by the quotients $\frac{\Sigma n q_c}{\Sigma n q_o}$. The values thus obtained will be called q'_o.

Having thus fixed the variation of the values of q with $\sin\lambda$, let us now inquire into the variation of these quantities with the value of p. The following tables embody the whole of the data at our disposal for that purpose.

TABLE 20.

$1.00 > \sin\lambda > 0.80$ (mean 0.94).

p	mean p	q'_o	q_c	O — C	n
° °	°				
0 to 15	7	1.20	1.18	+ 0.02	364
15 to 45	30	1.21	1.13	+ 0.08	539
45 to 75	60	0.95	1.03	— 0.08	220
75 to 105	90	0.81	0.89	— 0.08	158
105 to 135	120	0.80	0.78	+ 0.02	164
135 to 165	150	0.57	0.70	— 0.13	95
165 to 180	173	0.68	0.69	— 0.01	48

$0.80 > \sin\lambda > 0.60$ (mean 0.72).

p	mean p	q'_o	q_c	O — C	n
° °	°				
0 to 15	7	1.04	1.11	— 0.07	83
15 to 45	30	1.10	1.07	+ 0.03	160
45 to 75	60	0.96	1.00	— 0.04	102
75 to 105	90	0.97	0.89	+ 0.08	57
105 to 135	120	0.91	0.80	+ 0.11	37
135 to 165	150	0.72	0.75	— 0.03	39
165 to 180	173	0.67	0.72	— 0.05	14

$0.60 > \sin\lambda > 0.30$ (mean 0.48).

p	p	q'_o	q_c	O — C	n
° °	°				
0 to 15	7	0.90	1.03	— 0.13	60
15 to 45	30	0.96	1.01	— 0.05	111
45 to 75	60	0.94	0.95	— 0.01	85
75 to 105	90	1.18	0.89	+ 0.29	63
105 to 135	120	0.88	0.83	+ 0.05	49
135 to 165	150	0.58	0.79	— 0.21	42
165 to 180	173	0.94	0.78	+ 0.16	15

In the first group there seems to be a little systematic deviation in this sense that the positive residuals are somewhat in excess at the beginning, the negative ones somewhat in excess at the end. In the third group just the contrary seems to be the case. If now we take all three groups together and if we put:

$$\overline{R_1} = \frac{\Sigma n\,(O - C)}{\Sigma n} \text{ for the values } 7°,\ 30° \text{ and } 60°$$

$$\overline{R_2} = \frac{\Sigma n\,(O - C)}{\Sigma n} \text{ for the values } 120°,\ 150° \text{ and } 173°$$

we find:

$$\overline{R_1} = + 0.007 \qquad \overline{R_2} = - 0.024$$

therefore:

$$\overline{R_1} - \overline{R_2} = + 0.031.$$

We thus see by taking all three zones together, that the residuals show a little preference for positive values at the beginning for negative ones at the end.

We may now choose the value of h in such a way that we get:

$$\overline{R_1} - \overline{R_2} = 0.$$

After some trials it was found that the value

$$(g) \quad \ldots\ldots\ldots\ldots\ldots\ h = 0.750$$

satisfies this condition. Therefore the values of Q have been calculated afresh, taking this value of h as a basis. The value of C has also been calculated anew. The value now found is:

$$C = 0.884.$$

By multiplying the new values of Q with the new values of C, a new set of values q_c has been calculated. At the same time a new set of values of q_o has been formed by multiplying the values of q in every zone by the quotient $\frac{\Sigma n q_c}{\Sigma n q_o}$, using now for q_c the new values.

The comparison of the values of q_c and q'_o is shown in the following table.

TABLE 21

$1.00 > \sin\lambda > 0.80$ (mean 0.94)

p	mean p	q'_o	q_c	O–C	n
° °	°				
0 to 15	7	1.21	1.20	+ 0.01	364
15 to 45	30	1.22	1.16	+ 0.06	539
45 to 75	60	0.96	1.03	– 0.07	220
75 to 105	90	0.81	0.88	– 0.07	158
105 to 135	120	0.80	0.76	+ 0.04	164
135 to 165	150	0.57	0.68	– 0.11	95
165 to 180	173	0.68	0.67	+ 0.01	48

$0.80 > \sin\lambda > 0.60$ (mean 0.72).

p	mean p	q'_o	q_c	O – C	n
° °	°				
0 to 15	7	1.04	1.12	– 0.08	83
15 to 45	30	1.10	1.09	+ 0.01	160
45 to 75	60	0.96	1.00	– 0.04	102
75 to 105	90	0.97	0.88	+ 0.09	57
105 to 135	120	0.91	0.78	+ 0.13	37
135 to 165	150	0.72	0.73	– 0.01	39
165 to 180	173	0.67	0.70	– 0.03	14

$0.60 > \sin\lambda > 0.30$ (mean 0.48).

p	mean p	q'_o	q_c	O – C	n
° °	°				
0 to 15	7	0.89	1.03	— 0.14	60
15 to 45	30	0.96	1.02	— 0.06	111
45 to 75	60	0.94	0.95	— 0.01	85
75 to 105	90	1.17	0.88	+ 0.29	63
105 to 135	120	0.87	0.82	+ 0.05	49
135 to 165	150	0.58	0.77	— 0.19	42
165 to 180	173	0.93	0.76	+ 0.17	15

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The agreement has become much better. If we calculate again the values of $\overline{R_1}$ and $\overline{R_2}$ for all zones together we get:

$$\overline{R_1} = -0.002 \qquad \overline{R_2} = -0.002$$

therefore

$$\overline{R_1} - \overline{R_2} = 0.000$$

we may thus conclude that the agreement between the values of q_c and q'_o is satisfactory if we take for k the value (g).

Therefore this value of k, denoted by (15) in Chapter II, has been adopted for the calculation of the values of f in **Table II** (see page 26).

NOTE TO CHAPTER III.

To prove the formula for the proportions of the effects of a stardrift on the computed components ξ', η' and ζ of the solar velocity, defined in Chapter III, we will call a the angular value of the common velocity of the stars of the drift, directed towards the south pole, expressed in the unit of arc, the mean distance of the stars of the drift ρ. If the distances are very different we can divide the stars of the drift into groups for each of which the distance may be considered constant; the partial groups thus obtained must then be treated separately. We may however also treat all stars of the drift together, taking for ρ the mean distance of all stars of the group and for a a value which will be chosen in such a way that, whether we consider the real distances and angular drift motions, or imagine all the stars to be at distance ρ and to have the drift motion a, the effect on the components ξ, η, ζ (resp. ξ', η', ζ) of the sun's motion will be the same.

Now imagine the equations of BRAVAIS transformed in such a way that they are relative to the new system of coordinates defined on page 41 in which the components of the sun's motion are ξ', η', ζ.

Let:

A', B' and C be the coefficients of ξ', η' and ζ in the first, second and third equation respectively (the coefficient C remaining unchanged),

P', Q' and R the second members of the equations;

$\Delta P'$, $\Delta Q'$ and ΔR the effects of the star drift on the quantities P', Q' and R,

a' the right-ascension of a star counted from a point, which relatively to the original system of coordinates, has a right-ascension $\frac{a_1 + a_2}{2}$,

δ the mean of the declinations, which must not vary over too wide a range,

N the number of stars of the drift,

then, as a consequence of the supposition of a uniform distribution of the stars over the zone, made in Chapter III, we will have:

$$\Delta P' = \frac{N}{2\pi}\int_{-\frac{1}{2}\phi}^{+\frac{1}{2}\phi} \rho \sin\delta \cos a \,.\, a\, da = \frac{N}{2\pi}\rho \sin\delta \,.\, 2a \sin\tfrac{1}{2}\phi.$$

$$\Delta Q' = \frac{N}{2\pi}\int_{-\frac{1}{2}\phi}^{+\frac{1}{2}\phi} \rho \sin\delta \sin a \,.\, a\, da = 0.$$

$$\Delta R = -\frac{N}{2\pi}\int_{-\frac{1}{2}\phi}^{+\frac{1}{2}\phi} \rho \cos\delta \,.\, a\, \delta a = -\frac{N}{2\pi}\rho \cos\delta \,.\, a\phi.$$

$$\xi' = \frac{P'}{A'} \qquad \eta' = \frac{Q'}{B'} \qquad \zeta = \frac{R}{C}.$$

Therefore:

$$\Delta \xi' = \frac{N}{2\pi} \rho \sin \delta . \frac{2 a \sin \frac{1}{2} \phi}{A'}.$$

$$\Delta \eta' = 0.$$

$$\Delta \zeta = \frac{N}{2\pi} \rho \cos \delta . \frac{a \phi}{C}$$

$$\frac{\Delta \xi'}{\Delta \zeta} = \frac{2 \sin \frac{1}{2} \phi}{\phi} \frac{C}{A'} \operatorname{tg} \delta.$$

As a further consequence of the supposition of a uniform distribution we have

$$A' = A = B, \text{ therefore}$$

$$A' = \tfrac{1}{2} (A + B)$$

consequently

$$\frac{\Delta \xi'}{\Delta \zeta} = \frac{4 \sin \frac{1}{2} \phi}{\phi} \frac{C}{A + B} \operatorname{tg} \delta$$

$$\Delta \eta' = 0$$

which are the equations (43) used on page 42.

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EXPLANATION OF THE TABLES.

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TABLE I.

Newcomb — 20° to — 40°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	$\chi - \psi$
	h m	° ′		°	″	″	″	°	°	°	°
9	0 5·4	— 35 50	5·3	— 78	+ 0·117	+ 0·129	0·180	43	106	115	+ 72
34	30·9	— 25 27	5·8	— 87	+ 1·367	— 2	1·400	90	106	116	+ 26
58	52·6	— 30 2	4·5	— 86	— 23	— 13	0·026	240	111	112	— 128
109	1 39·8	— 25 41	5·6	— 77	+ 70	— 51	0·087	126	121	110	— 16
127	54·1	— 21 41	4·4	— 73	+ 114	— 8	120	95	122	112	+ 17
139	2 7·4	— 31 19	5·4	— 71	— 48	— 22	53	245	127	105	— 140
146	16·8	— 24 23	5·6	— 68	+ 188	— 76	200	112	128	110	— 2
179	43·8	— 32 56	4·4	— 63	+ 101	+ 156	190	32	136	100	+ 68
181	45·4	— 21 31	5·0	— 61	— 61	— 17	63	254	134	111	— 143
191	56·9	— 24 7	4·3	— 60	— 141	— 44	150	252	136	109	— 143
202	3 6·7	— 29 29	4·0	— 58	+ 314	+ 637	710	26	140	102	+ 76
222	28·2	— 22 3	4·5	— 52	+ 32	— 39	51	141	143	111	— 30
239	41·5	— 23 37	4·5	— 49	— 154	— 482	500	197	146	108	— 89
242	42·3	— 24 16	5·2	— 49	+ 50	+ 41	65	51	146	108	+ 57
243	44·8	— 36 35	4·3	— 50	— 43	— 28	51	237	148	87	— 150
247	48·9	— 35 6	5·3	— 50	+ 9	— 17	19	152	149	90	— 62
275	4 13·1	— 34 6	4·0	— 44	+ 31	0	31	90	154	88	— 2
279	19·3	— 34 18	4·1	— 43	+ 64	+ 43	77	56	155	87	+ 31
290	30·7	— 30 49	4·0	— 40	— 55	— 24	60	246	158	92	— 154
320	5 0·2	— 22 32	3·6	— 32	+ 17	— 64	66	165	163	116	— 49
335	13·0	— 35 1	5·2	— 32	+ 34	— 351	351	174	166	70	— 104
347	22·9	— 20 52	3·2	— 26		— 89	89	180	166	130	— 50
368	35·1	— 34 9	2·9	— 28	+ 7	0	39	170	171	62	— 108
370	39·2	— 22 29	4·0	— 23	— 295	— 38	460	220	169	137	— 83
378	45·9	— 20 53	4·1	— 21	+ 228	— 354	690	161	168	147	— 14
379	46·6	— 35 49	3·3	— 26	+ 31	— 648	390	4	171	41	+ 37
389	53·1	— 35 18	4·5	— 24	— 6	+ 389	8	315	173	37	+ 82
408	6 12·1	— 35 6	4·7	— 20	— 12	+ 6	66	350	173	0	+ 10
411	15·5	— 30 1	3·4	— 19	— 8	+ 65	24	199	176	343	+ 144
425	29·8	— 22 52	4·7	— 13	+ 30	— 23	46	41	170	211	+ 170
451	53·7	— 28 48	1·8	— 10	— 1	+ 35	3	342	170	262	— 80
453	56·7	— 27 45	3·7	— 9	— 8	+ 3	8	284	171	257	— 27
455	57·8	— 23 39	3·2	— 7	— 8	+ 2	10	302	169	238	— 64
460	7 3·3	— 26 12	2·2	— 7	— 20	+ 5	20	279	168	250	— 29
470	12·7	— 36 52	2·9	— 10	— 10	+ 3	14	225	165	295	+ 70
477	19·1	— 29 4	2·6	— 6	+ 4	— 10	8	30	166	265	— 125
502	44·0	— 24 33	3·6	+ 2	— 5	+ 7	5	270	159	250	— 20
519	59·2	— 39 39	2·5	— 4	— 51	0	51	263	156	286	+ 23
523	8 2·2	— 23 57	3·1	+ 5	— 88	— 6	102	301	155	251	— 50
557	38·6	— 32 44	3·9	+ 6	— 4	+ 52	12	340	149	267	— 73
596	9 16·0	— 25 26	5·2	+ 18	— 65	+ 11	73	244	139	253	+ 9
606	25·8	— 39 55	3·7	+ 9	— 207	— 32	210	281	140	270	— 11
622	38·6	— 27 12	5·3	+ 20	— 48	+ 38	56	301	135	254	— 47
664	10 21·4	— 30 26	4·6	+ 24	— 78	+ 29	81	254	127	254	0
696	50·9	— 36 28	4·8	+ 22	+ 136	— 23	200	135	123	255	+ 120
710	11 5·5	— 22 9	4·7	+ 35		— 138	110	180	114	246	+ 66
731	26·8	— 31 10	3·9	+ 29	— 204	0	210	255	114	249	— 6
762	12 3·7	— 21 55	3·4	+ 40	— 71	— 106	71	272	103	243	— 29
792	27·8	— 22 42	3·2	+ 39	— 10	+ 3	62	189	98	241	+ 52
813	46·5	— 39 30	4·5	+ 24	+ 70	— 61	78	117	102	241	+ 124
						— 35					

TABLE I.

Newcomb — 20° to — 40°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
833	13 9·9	— 30 51	5·4	+ 31	+ 0·017	— 0·071	0·073	167	95	238	+ 71
838	12·1	— 22 31	3·5	+ 39	+ 64	— 53	83	130	89	237	+ 107
839	13·6	— 36 3	3·2	+ 26	— 355	— 96	360	255	96	237	— 18
877	53·0	— 24 24	6·0	+ 36	— 215	— 103	240	244	82	233	— 11
882	59·2	— 26 5	3·7	+ 34	+ 42	— 146	160	165	82	232	+ 67
883	59·3	— 35 45	2·3	+ 24	— 529	— 523	740	225	88	231	+ 6
947	14 50·2	— 20 51	6·1	+ 32	+ 1·030	— 1·795	2·100	151	71	226	+ 75
953	56·8	— 24 47	3·6	+ 28	— 76	— 47	89	238	72	224	— 14
970	15 15·2	— 36 25	4·9	+ 17	— 51	— 55	75	223	78	219	— 4
1010	51·3	— 25 45	3·3	+ 19	— 13	— 48	50	195	66	214	+ 19
1012	52·9	— 22 16	2·7	+ 22	— 15	— 35	38	203	61	214	+ 11
1035	16 13·6	— 25 17	3·3	+ 17	— 15	— 39	42	201	62	209	+ 8
1044	18·1	— 23 9	4·9	+ 17	— 21	— 7	32	251	60	209	— 42
1051	21·7	— 26 9	1·5	+ 15	— 8	— 28	29	196	62	209	+ 13
1053	23·2	— 34 26	4·7	+ 9	— 9	— 28	29	198	68	205	+ 7
1061	28·1	— 27 57	3·1	+ 12	— 17	— 34	38	207	63	205	— 2
1073	42·1	— 34 4	2·5	+ 6	— 627	— 261	680	247	67	201	— 46
1075	43·4	— 37 50	3·5	+ 3	— 6	— 23	24	195	71	200	+ 5
1095	17 7·6	— 26 25	4·8	+ 7	— 502	— 1·167	1·200	204	58	197	— 7
1105	14·3	— 24 52	3·6	+ 6	— 8	— 36	37	193	57	196	+ 3
1109	18·7	— 24 3	4·5	+ 6	— 14	— 137	140	186	55	195	+ 9
1113	22·3	— 37 12	3·0	— 2	— 29	— 35	46	220	68	191	— 29
1118	25·1	— 37 1	2·0	— 2	— 5	— 27	27	191	68	191	0
1127	33·8	— 38 58	2·8	— 5	+ 5	— 15	16	162	69	188	+ 26
1158	57·8	— 30 25	3·2	— 5	— 73	— 198	210	199	60	184	— 15
1166	18 6·3	— 21 5	4·2	— 2	— 6	— 2	6	252	51	182	— 70
1169	9·2	— 36 48	3·3	— 10	— 131	— 152	200	221	67	181	— 40
1173	13·0	— 29 53	3·0	— 8	+ 30	— 34	46	139	60	180	+ 41
1175	15·9	— 34 26	2·1	— 10	— 51	— 122	130	203	64	180	— 23
1182	20·2	— 25 29	3·1	— 7	— 46	— 199	210	192	55	179	— 13
1199	37·8	— 27 7	3·5	— 11	+ 45	— 6	45	98	57	174	+ 76
1208	43·3	— 22 18	6·3	— 10	— 57	— 23	62	248	52	172	— 76
1211	47·5	— 26 27	2·3	— 13	— 4	— 75	75	183	56	170	— 13
1216	50·3	— 21 16	3·7	— 11	+ 31	— 23	39	127	51	170	+ 43
1222	54·6	— 30 3	2·9	— 16	— 31	— 19	36	238	60	169	— 69
1225	59·1	— 27 51	3·7	— 16	— 62	— 254	260	193	59	168	— 25
1228	19 1·0	— 38 6	4·4	— 20	+ 60	— 118	130	153	69	169	+ 16
1231	2·3	— 21 13	3·2	— 14	— 7	— 36	37	191	52	167	— 24
1237	7·9	— 25 28	5·1	— 17	+ 34	— 35	49	136	56	166	+ 30
1265	29·1	— 25 9	4·8	— 21	+ 60	— 27	66	114	58	161	+ 47
1280	39·1	— 20 4	5·3	— 21	— 140	— 88	170	237	54	156	— 81
1299	55·0	— 28 3	4·8	— 28	+ 30	+ 13	33	67	62	155	+ 88
1317	20 10·7	— 22 12	6·2	— 29	+ 17	— 32	36	152	59	150	— 2
1354	38·7	— 25 43	4·4	— 36	— 55	— 148	160	200	65	146	— 54
1363	44·3	— 27 23	4·6	— 37	— 11	— 6	13	241	67	145	— 96
1374	53·6	— 32 45	5·0	— 41	— 5	— 4	6	231	73	144	— 87
1376	55·0	— 39 7	5·7	— 42	— 47	— 125	130	202	78	145	— 57
1377	57·3	— 20 21	5·2	— 39	— 35	— 47	59	217	63	141	— 76
1386	21 5·9	— 28 8	5·8	— 43	+ 99	— 106	150	138	71	141	+ 3
1393	10·3	— 32 42	5·0	— 44	+ 33	— 10	34	107	72	140	+ 33

TABLE I.

Newcomb — 20° to — 40°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
1403	21 19·5	— 22 57	4·0	— 44	+ 0·006	+ 0·020	0·021	17	69	137	+ 120
1405	21·6	— 22 21	4·8	— 45	+ 126	— 18	130	99	68	137	+ 38
1423	37·5	— 33 36	4·6	— 50	+ 2	— 74	74	178	87	137	— 41
1434	46·3	— 37 57	3·4	— 52	+ 91	— 21	93	103	84	135	+ 32
1441	53·7	— 29 3	5·6	— 53	+ 21	+ 19	28	48	79	133	+ 85
1489	22 27·8	— 21 21	5·5	— 59	+ 206	— 155	260	126	80	127	+ 1
1497	33·7	— 27 42	4·4	— 62	+ 11	— 11	16	135	85	127	— 8
1516	50·7	— 30 17	1·5	— 66	+ 328	— 172	370	117	89	126	+ 9
1531	23 2·8	— 21 51	4·0	— 67	+ 44	+ 41	60	47	86	124	+ 77
1542	12·1	— 33 13	4·5	— 70	+ 1	— 66	66	179	94	122	— 57
1548	16·4	— 20 47	4·4	— 70	— 138	— 89	170	237	89	122	— 115
1558	26·2	— 38 31	4·8	— 71	+ 83	+ 6	83	86	100	121	+ 35
1560	26·7	— 21 36	4·9	— 73	— 22	+ 18	28	310	91	122	+ 172
1581	42·4	— 28 49	4·8	— 77	+ 77	— 133	150	150	97	120	— 30

Taylor — 20° to — 30°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
69	0 15·6	— 23 42	7·57	— 84	+ 0·078	— 0·098	0·125	141	102	117	— 24
194	36·4	— 20 53	7·04	— 83	— 12	+ 10	16	310	105	116	+ 166
492	1 26·2	— 24 18	7·39	— 79	— 46	— 70	84	213	117	112	— 101
527	31·2	— 25 39	8·92	— 78	— 94	+ 3	94	272	118	111	— 161
539	32·9	— 25 40	6.59	— 78	— 8	— 6	10	233	118	111	— 122
556	35·2	— 22 21	8·29	— 77	— 17	— 7	18	248	117	113	— 135
590	42·8	— 26 53	7·49	— 76	— 29	+ 19	35	303	121	110	+ 167
1076	3 6·6	— 29 38	7·09	— 58	+ 44	— 86	97	153	140	101	— 52
1306	42·3	— 29 51	7·35	— 50	+ 44	— 88	98	153	148	98	— 55
1390	57·3	— 28 52	7·72	— 47	— 71	— 47	85	236	151	100	— 136
1430	4 3·7	— 21 2	9·90	— 44	— 29	— 23	37	231	150	113	— 118
1605	29·0	— 28 43	7·05	— 40	+ 34	— 44	56	142	157	98	— 44
1667	39·5	— 28 11	7·53	— 38	— 66	— 7	66	264	159	100	— 164
1838	5 2·6	— 20 17	7·30	— 30	+ 35	— 64	73	151	161	125	— 26
2086	33·7	— 28 55	8·09	— 27	— 24	+ 58	63	338	172	96	+ 118
2310	6 1·6	— 23 5	6·74	— 18	— 17	+ 23	29	323	173	161	— 162
2360	6·5	— 27 2	7·82	— 19	+ 1	— 13	13	176	175	156	— 20
2435	15·4	— 29 58	7·96	— 18	0	+ 17	17	0	175	342	— 18
2658	40·7	— 20 29	8·39	— 9	— 13	— 11	17	230	167	214	— 16
2670	41·7	— 20 58	8·13	— 9	— 15	— 12	19	231	168	217	— 14
2733	48·6	— 24 5	7·48	— 9	+ 49	+ 32	59	57	169	234	+ 177
2780	54·2	— 28 48	8·40	— 10	— 17	— 4	17	257	170	268	+ 11
2862	7 4·6	— 25 0	5·76	— 6	+ 50	+ 6	50	82	167	246	+ 164
3093	28·8	— 28 18	6·78	— 3	— 58	+ 54	79	313	162	259	— 54
3238	43·6	— 24 39	8·93	+ 2	+ 248	— 244	350	134	159	251	+ 117
3342	54·6	— 23 1	8·33	+ 4	— 29	+ 12	31	293	156	249	— 44
3574	8 19·7	— 23 38	5·61	+ 9	— 71	+ 32	78	294	151	252	— 42
3986	9 4·2	— 25 18	7·30	+ 16	— 56	— 20	60	250	141	253	+ 3
4288	39·2	— 27 3	7·19	+ 20	— 33	— 36	49	222	134	254	+ 32
4582	10 13·3	— 28 20	9·23	+ 24	— 15	+ 10	18	304	128	253	— 51

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TABLE I.

Taylor — 20° to — 30°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
4618	10 16·6	— 28 56	8·01	+ 24	— 0·010	+ 0·058	0·059	350	127	253	— 97
4708	26·5	— 22 58	8·30	+ 30	— 61	— 31	69	243	123	249	+ 6
4827	39·1	— 29 2	7·97	+ 26	— 59	+ 26	64	294	122	251	— 43
5170	11 16·7	— 19 56	7·68	+ 38	— 66	— 26	71	248	111	245	— 3
5350	35·5	— 21 57	8·89	+ 38	— 17	+ 23	29	323	108	244	— 79
5506	56·6	— 21 27	8·88	+ 40	+ 74	+ 23	78	73	104	244	+ 171
5519	57·9	— 20 20	8·88	+ 41	+ 42	— 28	50	124	102	243	+ 119
5585	12 7·2	— 20 56	8·85	+ 41	+ 1	— 11	11	175	101	242	+ 67
5754	27·2	— 22 49	8·07	+ 39	+ 65	— 150	170	157	97	241	+ 84
6189	13 21·6	— 24 34	7·51	+ 37	— 65	— 16	67	256	89	237	— 19
6227	25·0	— 25 28	8·34	+ 37	— 117	— 11	120	265	88	235	— 30
6292	31·4	— 29 12	7·16	+ 32	+ 16	+ 53	55	17	89	235	— 142
6412	44·1	— 20 22	7·82	+ 40	— 72	— 37	81	243	82	234	— 9
6541	59·2	— 25 59	8·60	+ 34	+ 32	— 53	62	149	82	232	+ 83
6561	14 1·4	— 26 4	9·03	+ 33	+ 140	— 46	150	108	82	232	+ 124
6684	16·2	— 27 16	8·60	+ 31	— 245	— 278	370	221	80	228	+ 7
6691	16·8	— 27 11	9·03	+ 31	+ 15	+ 19	24	38	80	228	— 170
6744	23·3	— 28 45	8·03	+ 28	+ 7	— 9	11	142	80	228	+ 86
6812	30·9	— 22 37	7·76	+ 33	+ 10	0	10	90	75	227	+ 137
6820	32·2	— 24 29	8·26	+ 32	+ 72	— 44	84	122	76	228	+ 106
6853	35·9	— 24 35	8·46	+ 31	— 67	— 31	74	245	75	226	— 19
6935	48·2	— 25 7	8·60	+ 30	+ 5	— 11	12	156	73	225	+ 69
7060	15 3·9	— 21 36	8·66	+ 30	— 51	— 42	66	231	68	223	— 8
7074	6 5	— 23 54	8·18	+ 28	+ 39	— 22	45	119	70	222	+ 103
7189	21·1	— 20 46	7·84	+ 28	+ 47	— 32	57	124	65	221	+ 97
7216	24·6	— 27 45	7·99	+ 22	— 12	— 2	12	260	70	218	— 42
7297	35·7	— 25 1	7·29	+ 23	+ 5	— 20	21	166	66	216	+ 50
7433	57·6	— 20 34	8·35	+ 22	— 76	— 41	86	242	60	214	— 28
7452	59·3	— 27 23	8·37	+ 18	+ 29	— 46	54	148	65	211	+ 63
7499	16 5·1	— 27 48	8·93	+ 16	— 20	+ 48	52	337	65	210	— 127
7522	7·5	— 22 4	7·52	+ 20	+ 10	— 13	16	142	60	212	+ 70
7585	16·8	— 26 52	7·51	+ 15	— 89	— 62	108	235	63	208	— 27
7593	17·8	— 29 6	7·73	+ 14	+ 26	— 28	38	137	65	208	+ 71
7599	18·2	— 26 16	8·37	+ 16	— 1	— 29	29	182	62	209	+ 27
7717	36·3	— 22 53	7·76	+ 14	+ 40	— 68	79	150	59	204	+ 54
7930	17 3·9	— 26 33	8·08	+ 7	— 51	— 74	90	215	59	197	— 18
7935	4·8	— 26 52	8·41	+ 6	+ 1	— 17	17	177	59	197	+ 20
7951	7·5	— 27 49	8·95	+ 6	— 7	— 63	63	186	60	197	+ 11
7999	14·5	— 23 27	7·97	+ 7	— 4	+ 9	10	336	55	196	— 140
8009	15·5	— 24 59	7·44	+ 6	— 58	— 38	69	237	57	196	— 41
8028	18·3	— 27 29	7·59	+ 4	— 40	— 54	67	217	58	197	— 20
8037	19·2	— 21 18	8·99	+ 7	— 47	— 30	56	237	52	195	— 42
8066	23·2	— 23 45	7·53	+ 5	+ 14	— 65	66	168	55	193	+ 25
8090	26·1	— 22 5	8·53	+ 5	— 4	— 18	18	193	53	193	0
8137	33·2	— 23 46	7·86	+ 3	— 38	— 23	44	239	55	190	— 49
8171	37·2	— 21 58	9·22	+ 3	— 17	— 64	66	195	53	189	— 6
8181	38·2	— 26 55	7·56	+ 0	— 9	— 24	26	200	58	189	— 11
8249	47·4	— 22 57	8·41	+ 1	+ 51	— 19	55	111	53	186	+ 75
8355	18 0·2	— 21 52	8·99	— 1	— 4	— 22	22	190	52	183	— 7
8364	1·5	— 26 7	7·64	— 4	— 57	— 30	64	242	56	183	— 59

TABLE I.

Taylor — 20° to — 30°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	°	″	″	°	°	°	°
8376	18 2·5	— 23 47	7·81	— 3	— 0·029	— 0·052	0·060	209	54	183	— 26
8403	7·5	— 24 2	8·56	— 4	— 7	+ 2	7	286	54	182	— 104
8436	14·6	— 26 29	8·78	— 6	— 36	+ 4	36	276	56	180	— 96
8475	21·2	— 25 7	8·24	— 7	+ 58	— 43	72	127	55	178	+ 51
8492	22·8	— 24 9	8·35	— 7	— 13	— 8	15	238	54	177	— 61
8537	29·2	— 29 20	8·46	— 11	— 58	+ 6	58	276	59	176	— 100
8539	29·3	— 29 34	8·11	— 10	— 25	— 25	35	225	60	176	— 49
8613	41·4	— 20 17	8·13	— 9	— 48	— 31	57	237	50	171	— 66
8628	43·1	— 22 24	8·74	— 10	— 22	— 25	33	221	52	172	— 49
8665	49·0	— 23 18	8·11	— 12	— 3	— 86	86	182	53	170	— 12
8689	52·1	— 28 13	7·88	— 15	— 3	— 10	10	197	58	169	— 28
8698	53·8	— 23 24	8·35	— 13	+ 10	— 11	15	138	54	169	+ 31
8727	57·1	— 25 0	8·56	— 14	+ 41	— 7	42	100	56	169	+ 69
8738	58·5	— 27 28	7·34	— 16	— 7	+ 5	9	305	58	168	— 137
8783	19 3·9	— 20 33	7·93	— 14	+ 29	— 20	35	125	52	165	+ 40
8794	5·7	— 20 38	8·04	— 14	+ 51	— 17	54	109	52	165	+ 56
8796	6·1	— 22 47	8·38	— 16	— 79	+ 14	80	280	54	166	— 114
8926	22·3	— 21 36	8·38	— 18	+ 21	— 23	31	138	54	161	+ 23
9006	32·8	— 23 37	8·85	— 21	+ 10	— 22	24	156	57	160	+ 4
9158	53·1	— 21 12	8·15	— 25	— 11	— 4	12	250	56	153	— 97
9160	53·3	— 22 59	8·38	— 26	+ 14	+ 5	15	70	58	154	+ 84
9213	59·2	— 22 2	7·66	— 26	— 234	— 44	240	259	58	152	— 107
9230	20 2·3	— 20 57	7·22	— 27	— 24	— 81	84	196	57	152	— 44
9343	14·8	— 22 21	7·33	— 30	+ 6	— 23	24	165	59	150	— 15
9359	16·9	— 23 52	8·32	— 31	— 12	+ 25	28	334	62	149	+ 175
9422	24·8	— 25 18	7·37	— 33	+ 50	— 1	50	91	63	148	+ 57
9424	24·9	— 22 35	8·64	— 32	+ 3	+ 34	34	5	62	148	+ 143
9425	25·0	— 22 35	8·97	— 32	— 15	+ 37	40	338	62	148	+ 170
9478	31·2	— 21 26	6·79	— 33	+ 101	— 25	100	104	61	146	+ 42
9534	36·4	— 26 17	8·01	— 36	+ 103	+ 1	100	90	66	146	+ 56
9567	41·0	— 23 18	8·21	— 36	+ 36	+ 33	49	47	64	144	+ 97
9586	42·6	— 27 50	7·16	— 38	+ 42	— 17	45	112	68	146	+ 34
9629	47·0	— 26 3	7·69	— 38	0	— 70	70	180	67	143	— 37
9634	47·9	— 26 35	9.14	— 38	+ 31	— 2	31	94	68	144	+ 50
9673	52·8	— 27 50	8·09	— 40	+ 69	— 49	85	125	69	143	+ 18
9703	56·0	— 25 34	7·73	— 39	+ 48	— 21	52	114	68	142	+ 28
9771	21 3 0	— 29 0	8·03	— 42	+ 81	+ 18	83	77	71	142	+ 65
9788	4·9	— 20 50	7·60	— 40	+ 19	— 54	57	161	66	139	— 22
9810	7·9	— 21 18	7·80	— 41	— 107	— 110	160	225	66	139	— 86
9887	17·0	— 26 5	8·17	— 45	+ 27	— 36	45	143	71	138	— 5
9905	18·9	— 25 1	7·36	— 45	— 69	— 40	80	240	71	138	— 102
9908	19·3	— 21 32	7·80	— 44	+ 78	+ 14	79	80	68	137	+ 57
9922	21·1	— 22 15	8·51	— 45	+ 135	— 283	310	154	68	137	— 17
6944	23·2	— 24 58	8·67	— 46	+ 20	— 12	23	121	72	137	+ 16
9965	26·2	— 21 14	8·82	— 45	+ 14	+ 12	18	49	69	136	+ 87
9964	26·2	— 28 26	7·63	— 47	+ 215	— 51	220	104	74	138	+ 34
9968	26·9	— 28 27	8·67	— 47	+ 91	— 81	122	132	74	138	+ 6
9979	27·8	— 21 0	8·51	— 46	— 51	— 14	53	255	70	136	— 119
10011	31·2	— 28 27	8·17	— 48	— 8	— 23	24	199	75	137	— 62
10036	34·4	— 22 29	8·11	— 47	+ 120	+ 4	120	88	71	135	+ 47

OPCARD 101 V3

TABLE I.

Taylor — 20° to — 30°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
10038	21 34·5	− 22 14	8·31	− 47	+ 0·022	− 0 034	0·041	147	71	135	− 12
10106	42·1	− 23 24	8·30	− 49	− 7	0	7	270	73	134	− 136
10137	46·9	− 20 36	8·41	− 50	+ 18	+ 127	130	9	73	133	+ 124
10173	52·6	− 29 13	7·45	− 53	+ 21	+ 20	29	46	79	134	+ 88
10181	54·2	− 25 36	8·07	− 52	− 32	− 64	72	206	77	133	− 73
10196	56·0	− 27 39	8·04	− 53	− 17	− 13	21	233	78	133	− 100
10205	57·2	− 22 23	8·31	− 52	− 54	+ 14	56	285	74	132	− 153
10216	59·0	− 29 40	8·32	− 54	+ 19	− 24	31	142	81	132	− 10
10293	22 8·2	− 26 35	8·54	− 56	− 59	+ 3	59	273	79	131	− 142
10395	24·0	− 24 48	8.11	− 59	− 4	− 37	37	186	81	129	− 57
10440	30·0	− 21 44	8·18	− 60	− 28	+ 24	37	311	80	127	+ 176
10445	30·7	− 21 2	7·30	− 60	+ 82	− 44	93	118	80	127	+ 9
10471	34·6	− 27 55	8·54	− 62	− 30	+ 5	30	280	86	127	− 153
10518	41·6	− 20 22	8·95	− 62	− 102	− 159	190	212	82	126	− 86
10577	53·6	− 26 48	9·08	− 66	− 8	+ 14	16	330	88	125	+ 155
10655	23 7 8	− 29 8	8·70	− 69	− 52	0	52	270	91	123	− 147
10678	11·8	− 29 10	8·16	− 70	+ 31	− 42	52	144	92	122	− 22
10729	19 3	− 21 18	8·63	− 70	− 60	− 192	200	197	89	122	− 75
10735	19·9	− 21 53	9·20	− 71	+ 42	+ 34	54	51	90	122	+ 71
10796	30·1	− 22 56	7·89	− 74	+ 36	+ 27	45	53	93	121	+ 68
10809	32·2	− 21 34	8·17	− 74	− 35	+ 39	53	318	92	121	+ 163
10838	38·0	− 26 57	6·43	− 76	− 77	− 3	77	268	96	120	− 148
10857	40·9	− 22 58	7·82	− 76	− 8	− 95	95	185	95	120	− 65

Taylor — 30° to — 35°.

951	2 44·0	− 32 54	9·07	− 63	+ 0·087	+ 0·003	0·087	88	135	100	+ 12
952	44·1	− 31 20	7·74	− 63	+ 18	0	18	90	135	102	+ 12
1632	4 32·9	− 31 40	8·00	− 40	− 22	0	22	270	158	90	+ 180
2460	6 18·0	− 33 48	8·84	− 19	− 56	+ 1	56	271	173	344	+ 73
2585	32·4	− 32 7	7·96	− 16	+ 323	− 62	330	101	173	300	− 161
2708	45·7	− 31 34	9·13	− 13	− 5	− 20	21	194	171	287	+ 93
3047	7 22·6	− 33 38	7·42	− 7	+ 42	− 9	43	102	164	281	+ 179
3471	8 8·3	− 31 47	8·24	+ 2	+ 13	− 49	51	165	155	268	+ 103
4089	9 15·4	− 31 14	6·95	+ 14	− 15	− 38	41	202	142	260	+ 58
4472	10 0·0	− 34 16	6·90	+ 18	+ 48	+ 1	48	89	132	259	+ 170
4685	23·8	− 32 46	8·32	+ 22	− 44	− 25	51	240	127	256	+ 16
4716	27·2	− 32 44	8·64	+ 22	− 31	− 53	61	210	127	255	+ 45
4735	29·0	− 33 7	8·76	+ 22	− 24	− 13	27	241	126	255	+ 14
4882	44·6	− 33 11	7·79	+ 24	− 111	− 33	120	252	123	254	+ 2
5063	11 4·6	− 31 53	8·55	+ 26	− 5	− 41	41	187	118	251	+ 64
5261	26·3	− 30 17	8·95	+ 30	− 87	− 17	89	259	114	249	− 10
5286	29·6	− 34 6	8·31	+ 26	− 7	+ 13	15	332	114	250	− 82
5300	30·5	− 33 57	7·72	+ 26	− 278	+ 88	290	288	114	249	− 39
5331	33·5	− 34 16	8·18	+ 27	− 27	− 17	32	237	113	249	+ 12
5352	35·5	− 31 48	5·57	+ 28	− 28	− 65	71	203	112	248	+ 45

TABLE I.

Taylor — 30° to — 35°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
5572	12 5·2	− 29 54	7·70	+ 32	− 0·079	+ 0·017	0·081	282	106	244	− 38
5693	20·8	− 32 5	8·63	+ 30	− 80	− 6	80	266	104	244	− 22
5952	52·0	− 32 43	8·98	+ 30	− 39	− 39	55	225	98	240	+ 15
5977	56·2	− 33 37	7·80	+ 29	+ 5	− 46	46	174	98	240	+ 66
6007	13 0·1	− 33 27	7·39	+ 29	− 5	− 83	83	183	96	239	+ 56
6908	14 45·2	− 33 7	8·03	+ 23	+ 38	− 18	42	115	80	225	+ 110
7547	16 11·6	− 30 36	5·47	+ 13	+ 72	− 32	79	114	67	208	+ 94
7750	41·2	− 33 47	7·25	+ 6	− 30	− 60	67	207	67	201	− 6
7854	54·4	− 33 5	7·50	+ 6	+ 29	− 119	120	166	64	199	+ 33
8139	17 33·5	− 33 3	8·08	− 1	+ 75	− 13	76	100	62	189	+ 89
8352	18 0·0	− 31 1	8·11	− 6	− 31	− 3	31	264	61	183	− 81
8402	7·5	− 33 8	7·04	− 8	+ 13	− 20	24	147	63	183	+ 36
8479	21·8	− 33 37	8·24	− 11	− 20	− 6	21	253	64	170	− 75
8480	21·8	− 33 34	8·24	− 11	− 20	− 15	25	233	64	178	− 55
8499	24·2	− 33 2	7·52	− 11	− 3	− 11	11	195	63	177	− 18
8517	25·8	− 33 3	7·05	− 12	− 9	+ 14	17	327	63	177	− 150
8559	33·0	− 30 38	7·98	− 12	− 13	− 61	62	192	61	174	− 18
9038	19 37·1	− 32 5	7·84	− 25	+ 62	− 11	63	100	65	160	+ 60
9636	20 48·1	− 32 2	7·80	− 40	+ 18	− 48	51	159	72	145	− 14
9657	50·8	− 32 11	8·46	− 41	+ 53	− 49	72	133	72	145	+ 12
9717	57·5	− 32 51	8·46	− 42	− 10	+ 4	11	292	73	144	− 148
10400	22 24·4	− 33 0	4·48	− 60	+ 83	− 23	86	106	87	129	+ 23
10438	29·6	− 32 18	7·48	− 61	− 81	− 28	86	251	87	128	− 123
10820	34·5	− 32 46	7·35	− 62	+ 82	+ 52	97	58	89	127	+ 69
10915	50·0	− 30 12	7·89	− 65	− 4	− 52	52	184	89	126	− 58

Newcomb — 40° to — 52°.

21	0 20·1	− 42 59	2·6	− 74	+ 0·231	− 0·403	0·460	150	111	111	− 39
91	1 22·9	− 43 58	3·5	− 71	− 5	− 225	230	181	122	102	− 79
187	2 53·5	− 40 48	3·2	− 60	− 2	+ 24	24	355	139	90	+ 95
216	3 21·7	− 42 5	6·7	− 54	+ 16	+ 7	17	66	143	84	+ 18
297	4 36·5	− 42 6	4·8	− 40	− 138	− 107	180	232	157	63	− 169
390	5 55·3	− 42 49	4·2	− 26	+ 70	− 42	82	121	166	15	− 106
395	6 0·9	− 45 2	6·6	− 26	− 68	+ 224	230	342	164	10	+ 28
429	6 33·9	− 43 5	3·4	− 19	+ 36	− 19	41	118	165	340	− 138
483	7 25·3	− 43 3	3·2	− 11	− 52	+ 180	190	344	159	307	− 37
508	7 47·9	− 40 15	3·9	− 6	+ 5	+ 11	12	24	158	291	− 93
525	8 5·7	− 46 58	2·1	− 7	+ 24	− 11	27	115	152	299	− 176
553	8 36·5	− 46 12	3·9	− 2	− 48	− 27	55	240	147	288	+ 48
585	9 3·4	− 42 56	2·5	+ 4	+ 10	− 7	12	125	143	277	+ 152
645	10 9·5	− 41 30	4·2	+ 13	− 144	+ 32	150	281	132	265	− 16
675	10 32·1	− 47 35	4·2	+ 9	− 157	− 52	170	253	128	266	+ 13
760	12 1·9	− 50 2	3·0	+ 12	− 22	− 30	37	216	114	252	+ 36
800	12 34·6	− 48 16	2·6	+ 15	− 169	− 19	170	263	108	245	− 18
864	13 42·0	− 41 4	3·7	+ 20	− 51	− 22	56	247	94	234	− 13
865	13 42·1	− 41 51	3·5	+ 19	− 8	− 21	22	201	95	234	+ 33
871	13 47·8	− 46 40	3·0	+ 14	− 45	− 64	78	215	97	233	+ 18

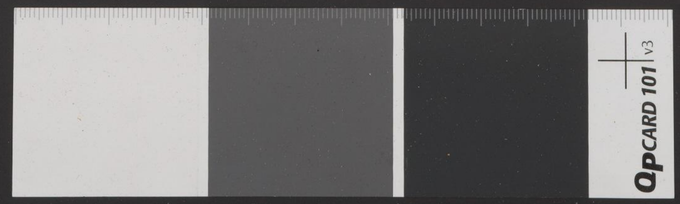

TABLE I.

Newcomb — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	$\chi - \psi$
	h m	° ′		°	″	″	″	°	°	°	°
903	14 18·1	— 44 39	4.8	+ 15	— 0·008	— 0·030	0·031	195	91	228	+ 33
914	14 27·6	— 41 36	2·9	+ 17	— 10	— 32	34	197	88	226	+ 29
920	14 33·6	— 46 51	2·7	+ 11	— 16	— 35	38	205	91	224	+ 19
948	14 50·3	— 42 38	2·9	+ 14	— 50	— 62	80	219	86	223	+ 4
949	14 51·0	— 41 36	3·5	+ 15	+ 30	— 26	40	130	85	223	+ 93
958	15 3·3	— 48 16	4·4	+ 8	— 97	— 62	115	237	88	219	— 18
959	15 3·3	— 51 37	3·7	+ 5	— 91	— 65	112	235	91	219	— 16
969	15 13·2	— 40 12	3·6	+ 14	+ 39	— 24	46	122	81	219	+ 97
984	15 26·8	— 40 45	3·2	+ 12	+ 4	— 49	49	175	81	216	+ 41
1032	16 10·5	— 49 51	4·4	0	— 184	— 62	190	252	84	206	— 46
1093	17 3·2	— 43 4	3·6	— 3	+ 50	— 306	310	171	74	195	+ 24
1115	17 22·2	— 49 46	3·1	— 9	— 9	— 83	83	186	80	190	+ 4
1121	17 28·3	— 42 55	2·2	— 5	+ 16	— 9	18	119	73	189	+ 70
1135	17 38·8	— 40 5	3·3	— 6	+ 32	— 8	33	104	70	188	+ 84
1162	18 1·9	— 45 58	4·7	— 13	— 43	— 45	62	224	76	183	— 41
1245	19 13·6	— 44 41	4·3	— 24	+ 20	— 24	31	140	76	167	+ 27
1396	21 12·7	— 41 20	5·2	— 45	+ 57	+ 5	57	85	82	142	+ 57
1451	22 0·3	— 47 34	2·1	— 53	+ 137	— 175	220	141	92	133	— 8

Gill — 40° to — 52°.

4	23 59·5	— 42 27	7·4	— 73	+ 0·092	— 0·090	0·129	134	107	114	— 20
25	0 3·1	— 46 26	4·02	— 69	+ 129	— 190	230	146	110	112	— 34
38	5·6	— 42 52	6·2	— 73	— 105	— 60	120	240	108	113	— 127
78	12·5	— 43 56	6·52	— 73	+ 48	00	48	90	110	112	+ 22
110	18·6	— 51 44	6·8	— 65	+ 534	— 250	590	115	114	109	— 6
119	20·0	— 44 25	4·12	— 72	+ 101	+ 30	100	73	111	111	+ 38
131	22·7	— 51 13	6·2	— 66	+ 139	— 20	140	98	114	108	+ 10
138	23·2	— 41 21	6·8	— 76	+ 4	— 30	30	172	111	111	— 61
142	24·3	— 41 38	6·2	— 75	— 8	+ 20	22	338	111	111	+ 133
143	24·3	— 40 12	6·85	— 77	+ 37	— 80	88	155	110	111	— 44
144	24·4	— 48 54	5·64	— 68	+ 115	— 120	160	137	114	108	— 29
165	28·3	— 43 7	6·8	— 74	+ 80	+ 20	83	76	113	110	+ 34
173	29·7	— 48 41	5·62	— 69	+ 46	— 130	140	161	115	108	— 53
197	33·9	— 45 29	6·2	— 72	+ 26	— 10	28	111	114	107	— 4
201	34·9	— 41 13	8·3	— 76	— 30	— 20	36	236	113	110	— 126
203	35·4	— 46 46	4·78	— 70	— 15	— 20	25	217	115	107	— 110
225	39·0	— 43 21	6·2	— 74	— 94	— 120	150	218	115	108	— 110
228	39·5	— 49 31	7·4	— 68	+ 46	— 10	47	102	116	105	+ 3
235	39·9	— 48 14	5·88	— 69	+ 186	+ 70	200	70	116	106	+ 36
246	43·1	— 47 23	6·28	— 70	— 24	+ 20	31	310	116	106	+ 156
253	44·2	— 44 5	6·48	— 73	— 6	00	6	270	115	108	— 162
262	45·0	— 51 40	5·17	— 65	+ 63	+ 30	70	64	118	104	+ 40
304	53·2	— 51 56	8·0	— 65	+ 54	00	54	90	119	102	+ 12
330	57·2	— 47 4	5·35	— 70	+ 6	00	6	90	118	104	+ 14
347	1 0·5	— 47 23	3·56	— 69	— 24	+ 10	26	293	119	103	+ 170

TABLE I.

Gill — 40° to — 52°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	$\chi - \psi$
	h m	° ′		°	″	″	″	°	°	°	°
355	1 1·8	— 42 25	6·8	— 74	+ 0·169	— 0·090	0·190	118	118	106	— 12
418	13·2	— 43 59	6·8	— 72	— 17	+ 10	20	300	120	107	+ 167
435	16·1	— 45 48	7·1	— 70	+ 151	+ 50	160	72	121	101	+ 29
440	16·9	— 44 15	6·2	— 72	+ 102	+ 40	110	68	121	102	+ 34
446	18·1	— 41 36	6·8	— 74	— 29	— 90	95	198	120	103	— 95
452	19·3	— 45 11	6·49	— 71	— 6	— 30	31	191	121	101	— 90
484	26·0	— 49 43	4·09	— 66	+ 133	+ 140	190	43	124	98	+ 55
485	26·1	— 50 33	7·2	— 65	— 40	— 80	89	207	125	98	— 109
487	26·3	— 46 13	5·7	— 69	— 15	0	15	270	123	100	— 170
504	29·0	— 46 20	7·7	— 69	+ 119	+ 90	150	53	124	99	+ 46
516	31·7	— 49 27	6·8	— 66	— 43	+ 20	47	295	124	97	+ 167
522	32·8	— 46 43	7·1	— 68	+ 36	— 10	37	106	123	98	— 8
526	33·2	— 43 34	6.8	— 71	— 71	— 30	77	247	123	100	— 147
542	36·7	— 50 40	6·9	— 65	— 21	— 20	29	226	126	106	— 120
564	41·2	— 51 26	5·57	— 63	+ 45	— 30	54	124	127	93	— 31
571	42·0	— 42 23	6·2	— 71	+ 15	+ 30	34	27	124	100	+ 73
587	45·3	— 48 26	5·7	— 65	+ 136	+ 50	150	70	126	96	+ 26
592	46·1	— 40 27	6·2	— 71	+ 26	— 40	48	147	125	101	— 46
607	48·6	— 46 55	4·28	— 66	— 86	— 90	124	224	127	95	— 129
609	49·2	— 43 7	5·12	— 69	— 39	— 50	63	218	126	98	— 120
629	52·2	— 48 0	4·82	— 65	+ 116	+ 20	120	80	128	94	+ 14
632	53·2	— 41 47	6·8	— 69	+ 48	— 30	57	122	127	98	— 24
642	54·5	— 42 38	5·52	— 68	— 51	— 110	120	204	127	98	— 106
652	56·7	— 45 19	5·08	— 67	— 16	— 50	52	198	127	96	— 102
685	2 3·0	— 42 28	6·2	— 68	— 18	— 30	35	211	129	97	— 114
695	4·7	— 41 27	6·2	— 68	+ 48	— 30	57	122	130	97	— 25
698	5·1	— 44 24	6·2	— 66	+ 47	— 20	51	113	130	95	— 18
701	5·5	— 51 26	6·2	— 61	+ 2·150	+ 660	2·200	71	130	89	+ 18
754	15·5	— 44 38	8·0	— 65	+ 5	+ 10	12	27	131	94	+ 67
766	17·3	— 43 46	6·5	— 65	+ 91	+ 50	104	61	132	94	+ 33
774	18·5	— 51 40	5·7	— 60	— 20	+ 70	73	344	132	86	+ 102
780	19·5	— 41 25	5·7	— 65	+ 272	+ 120	300	66	132	95	+ 29
795	22·4	— 48 16	4·45	— 61	+ 16	— 20	26	141	132	88	— 53
823	27.6	— 46 25	7·6	— 62	— 46	— 20	50	246	133	90	— 156
835	29·6	— 51 39	6·2	— 58	+ 7	— 30	31	167	134	84	— 83
865	35·0	— 43 26	4·89	— 62	+ 80	— 60	100	127	135	91	— 36
869	35·7	— 40 23	4·09	— 63	+ 129	— 40	140	107	134	94	— 13
884	38·3	— 51 20	5·48	— 57	+ 308	+ 220	380	55	135	82	+ 27
888	38·5	— 41 4	6·2	— 62	+ 15	+ 10	18	56	134	93	+ 37
901	40·8	— 46 49	6·8	— 60	— 5	— 30	30	189	136	85	— 104
902	40·8	— 43 22	6·8	— 61	+ 26	— 60	65	157	136	90	— 67
922	44·4	— 46 52	7·8	— 59	+ 36	— 20	41	119	136	85	— 34
927	45·6	— 40 27	6·49	— 62	+ 83	— 10	84	97	135	93	— 4
941	48·0	— 41 54	6·2	— 60	+ 48	— 30	57	122	136	90	— 32
982	55·5	— 42 22	7·0	— 59	+ 4	— 30	30	172	138	89	— 83
995	57·6	— 48 3	7·4	— 56	— 64	0	64	270	139	81	+ 171
1000	58·7	— 47 28	5·84	— 56	+ 36	— 10	37	106	139	82	— 24
1010	3 0·6	— 51 49	7·4	— 54	+ 100	+ 80	130	51	138	76	+ 25
1040	6·4	— 49 12	6·16	— 55	+ 36	— 30	47	130	140	78	— 52
1048	8·0	— 44 53	6·04	— 56	+ 111	— 20	110	100	140	83	— 17

TABLE I.

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
1054	3 8·7	— 41 51	6·8	— 56	— 0·018	— 0·010	0·021	241	140	87	— 154
1066	10·7	— 41 42	8·0	— 56	— 7	+ 10	12	325	141	86	+ 121
1079	13·4	— 48 13	6·01	— 54	+ 6	+ 30	31	11	141	78	+ 67
1087	15·0	— 43 33	4·44	— 55	+ 3·045	+ 740	3·100	77	142	84	+ 7
1122	20·9	— 51 30	6·2	— 52	+ 7	+ 30	31	13	141	73	+ 60
1139	23·4	— 44 17	7·1	— 53	+ 47	+ 20	51	67	143	81	+ 14
1153	26·6	— 47 48	6·11	— 52	+ 96	0	96	90	143	76	— 14
1167	28·9	— 50 48	5·76	— 51	+ 82	+ 80	115	46	144	70	+ 24
1181	31·6	— 44 8	6·9	— 52	+ 37	+ 10	38	75	144	79	+ 4
1188	32·6	— 40 41	4·65	— 53	— 8	— 40	41	191	145	83	— 108
1212	36·7	— 41 10	7·1	— 52	— 8	— 20	22	202	146	82	— 120
1221	38·2	— 46 22	6·4	— 50	— 67	0	67	270	145	73	+ 163
1232	39·7	— 41 3	6·2	— 51	+ 37	— 100	110	160	147	82	— 78
1239	41·4	— 47 45	5·77	— 50	— 4	— 20	20	191	145	72	— 119
1267	45·6	— 43 6	7·0	— 49	— 18	— 40	44	204	147	78	— 126
1291	49·1	— 41 36	7·1	— 49	+ 4	+ 60	60	4	147	80	+ 76
1292	49·7	— 47 16	7·2	— 48	+ 36	— 50	62	145	147	70	— 75
1295	50·0	— 40 44	5·72	— 49	+ 15	— 30	34	153	149	79	— 74
1297	50·7	— 46 47	7·1	— 48	— 5	— 30	30	189	147	70	— 119
1322	54·7	— 49 58	7·27	— 47	+ 36	+ 20	41	61	147	65	+ 4
1368	4 3·3	— 49 58	7·41	— 46	+ 55	+ 10	56	80	148	62	— 18
1379	4·7	— 46 12	6·39	— 46	+ 77	+ 10	78	83	150	67	— 16
1399	8·5	— 44 41	6·90	— 45	+ 37	0	37	90	151	68	— 22
1406	9·3	— 40 41	6·2	— 45	+ 15	0	15	90	152	76	— 14
1410	9·8	— 42 36	3·96	— 45	+ 59	— 230	240	165	152	72	— 93
1441	15·3	— 44 34	5·22	— 44	+ 69	— 50	85	126	153	67	— 59
1468	20·6	— 46 56	7·1	— 43	— 15	— 20	25	217	152	60	— 157
1469	20·6	— 44 19	8·3	— 43	+ 5	— 10	11	153	153	66	— 87
1474	21·4	— 44 27	6·8	— 43	+ 15	+ 60	62	14	153	65	+ 51
1490	23·4	— 47 13	6·39	— 42	+ 67	— 290	300	166	153	58	— 108
1496	24·0	— 42 14	7·1	— 43	— 29	— 10	31	251	155	67	+ 176
1500	25·6	— 46 48	6·49	— 42	+ 36	+ 20	41	61	153	58	— 3
1509	27·0	— 45 14	5·45	— 42	— 6	— 10	12	211	154	60	— 151
1541	33·3	— 42 8	6·2	— 41	+ 37	+ 40	55	43	156	64	+ 21
1561	37·2	— 48 47	6·8	— 40	+ 6	+ 20	21	17	154	52	+ 35
1574	39·6	— 50 43	5·45	— 40	— 31	+ 40	51	322	154	47	+ 85
1578	39·6	— 41 18	6·8	— 40	+ 15	+ 10	18	56	157	63	+ 7
1607	44·7	— 44 12	6·8	— 39	+ 26	+ 20	33	52	157	57	+ 5
1632	48·4	— 51 56	6·5	— 38	— 11	— 10	15	228	152	42	+ 174
1667	56·2	— 49 39	7·1	— 37	+ 16	+ 10	19	58	155	43	— 15
1693	59·5	— 49 20	5·61	— 37	— 33	0	33	270	156	42	+ 132
1701	5 1·7	— 49 45	5·08	— 37	+ 36	0	36	90	156	41	— 49
1721	4·5	— 41 23	7·1	— 35	— 86	+ 280	290	343	162	55	+ 72
1727	5·9	— 44 29	7·5	— 36	+ 26	+ 110	110	13	160	48	+ 35
1789	16·6	— 47 10	7·2	— 34	— 5	— 10	11	207	159	37	— 170
1801	17·8	— 44 30	8·3	— 33	— 27	+ 10	29	291	161	42	+ 110
1804	18·4	— 51 42	7·0	— 33	+ 26	+ 30	40	41	156	30	— 11
1821	21·2	— 44 20	6·5	— 33	— 6	— 20	21	197	161	39	— 158
1833	23·5	— 44 58	7·39	— 32	+ 5	+ 40	40	7	161	36	+ 29
1853	26·4	— 47 10	6·9	— 32	+ 36	— 40	54	138	160	31	— 107

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	°	″	″	°	°	°	°
1887	5 32·7	— 47 23	6·8	— 31	— 0·045	— 0·040	0·060	228	160	27	+ 159
1938	40·1	— 45 53	6·8	— 30	+ 26	+ 80	84	18	163	23	+ 5
1950	43·0	— 46 38	5·29	— 28	— 36	+ 10	37	286	162	22	+ 96
1956	43·5	— 41 38	6·8	— 28	— 7	— 30	31	193	167	27	— 166
1994	49·9	— 47 59	7·4	— 28	— 14	— 30	33	205	160	15	+ 170
2027	56·1	— 51 14	7·4	— 28	— 2	+ 10	10	349	158	10	+ 21
2059	6 1·0	— 51 5	7·8	— 27	+ 7	0	7	90	158	7	— 83
2062	1·5	— 48 27	6·8	— 27	— 114	— 40	120	251	161	7	+ 116
2066	2·6	— 45 48	7·7	— 25	+ 36	— 70	79	153	164	7	— 146
2069	3·4	— 45 5	6·51	— 25	+ 79	+ 30	85	69	164	8	— 61
2071	3·5	— 46 11	7·5	— 25	— 98	— 30	102	253	163	7	+ 114
2077	4·0	— 42 8	5·70	— 24	— 29	— 30	42	224	167	8	+ 144
2078	4·1	— 44 43	7·03	— 25	— 17	+ 20	26	320	164	7	+ 47
2092	6·1	— 40 20	5·70	— 24	— 20	+ 50	54	338	169	7	+ 29
2096	7·1	— 45 15	6·37	— 25	— 16	— 20	26	219	164	4	+ 145
2147	15·7	— 50 18	7·05	— 25	+ 7	+ 10	12	35	159	357	— 38
2190	22·4	— 48 6	6·15	— 23	— 14	— 30	33	205	160	353	+ 148
2201	23·8	— 41 3	8·0	— 21	+ 15	+ 90	91	9	167	347	— 22
2207	25·1	— 40 17	8·0	— 20	+ 26	0	26	90	167	345	— 105
2226	28·4	— 51 44	5·85	— 23	+ 100	+ 90	130	48	157	351	— 57
2229	28·7	— 45 13	7·37	— 21	+ 26	— 40	48	147	163	346	— 161
2244	30·2	— 42 0	7·5	— 20	— 18	— 60	63	197	166	342	+ 145
2258	33·4	— 50 13	7·41	— 22	+ 26	+ 140	140	10	158	347	— 23
2269	34·5	— 49 25	6·9	— 21	— 72	+ 20	75	286	159	346	+ 60
2285	37·2	— 40 11	8·12	— 18	— 8	— 40	41	191	166	332	+ 141
2286	37·4	— 47 30	7·1	— 20	— 4	0	4	270	161	343	+ 73
2323	43·4	— 47 40	7·4	— 20	+ 26	— 10	28	111	160	338	— 133
2331	44·5	— 43 40	7·4	— 18	— 6	+ 50	50	353	165	331	— 22
2348	46·9	— 50 28	2·93	— 20	+ 36	— 80	88	156	158	339	— 177
2359	48·7	— 42 21	7·6	— 17	— 7	— 20	21	199	165	326	+ 127
2371	50·5	— 42 13	6·2	— 16	+ 37	+ 10	38	75	165	325	— 110
2375	51·0	— 50 28	7·1	— 19	— 40	+ 210	210	349	158	336	— 13
2385	53·0	— 48 34	5·08	— 18	+ 6	+ 10	12	31	160	333	— 58
2412	57·1	— 48 58	7·1	— 18	+ 6	— 30	31	169	159	332	+ 163
2421	58·3	— 43 13	6·8	— 15	+ 4	— 40	40	174	164	322	+ 148
2442	7 2·0	— 42 8	6·8	— 15	— 29	+ 10	31	289	164	317	+ 28
2458	4·2	— 51 46	6·21	— 18	— 21	+ 50	54	337	156	331	— 6
2471	6·1	— 40 10	8·7	— 13	— 54	— 20	58	250	165	310	+ 60
2490	9·0	— 46 33	4·71	— 15	— 147	+ 90	170	301	160	321	+ 20
2497	9·5	— 44 58	5·25	— 14	— 6	— 120	120	183	161	320	+ 137
2500	9·7	— 44 26	5·4	— 14	+ 112	+ 320	340	19	162	317	— 62
2509	11·2	— 46 38	6·01	— 15	— 15	— 30	34	207	160	320	+ 113
2524	13·9	— 46 46	7·4	— 14	— 35	+ 560	560	357	159	318	— 39
2534	15·9	— 46 59	7·7	— 14	— 45	— 10	46	257	158	319	+ 62
2577	23·2	— 50 46	5·21	— 15	— 12	0	12	270	155	320	+ 50
2605	27·2	— 50 21	6·9	— 14	— 2	+ 10	10	349	155	318	— 31
2624	30·2	— 50 19	6·96	— 13	— 22	0	22	270	154	318	+ 48
2631	30·9	— 44 1	6·8	— 11	+ 4	0	4	90	158	306	— 144
2643	33·2	— 48 33	6·08	— 12	— 24	0	24	270	155	314	+ 44
2651	34·8	— 48 19	5·78	— 12	— 14	0	14	270	155	312	+ 42

TABLE I.

Gill — 40° to — 52°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
2662	7 35·8	— 48 46	7·0	— 12	+ 0·006	+ 0·030	0·031	11	155	313	— 58
2679	39·0	— 44 51	5·25	— 10	— 59	— 570	570	186	156	305	+ 119
2683	39.5	— 40 38	5·27	— 7	+ 151	— 190	240	142	158	294	+ 152
2710	43·1	— 47 48	7·6	— 10	— 14	+ 10	17	306	155	308	+ 2
2715	43·7	— 46 18	5·45	— 9	— 5	— 10	11	207	156	304	+ 97
2730	45·4	— 46 3	4·28	— 9	— 5	— 10	11	207	156	304	+ 97
2734	47·0	— 50 11	6·02	— 11	— 32	— 50	59	212	152	310	+ 98
2755	49·5	— 49 17	4·98	— 10	+ 16	+ 10	19	58	153	308	— 110
2782	53·9	— 45 14	5·31	— 7	— 6	+ 20	21	343	155	299	— 44
2813	58·4	— 44 19	7·2	— 6	— 6	0	6	270	154	296	+ 26
2823	59·9	— 49 9	7·1	— 9	— 43	+ 10	44	283	152	304	+ 21
2828	8 1·2	— 50 14	6·21	— 9	— 22	— 10	24	245	151	305	+ 60
2834	1·5	— 46 38	7·0	— 7	— 25	0	25	270	153	299	+ 29
2846	4·0	— 48 39	6·8	— 8	+ 46	— 10	27	102	151	301	— 161
2860	5·9	— 47 34	5·55	— 7	+ 6	— 20	21	163	152	299	+ 136
2925	15·2	— 47 49	7·1	— 6	— 4	— 20	20	191	151	297	+ 106
2935	16·5	— 51 33	7·1	— 8	— 30	+ 50	58	329	148	301	— 28
2947	18·7	— 48 6	5·00	— 5	+ 6	— 10	12	149	149	297	+ 148
2974	22·8	— 49 5	7·1	— 6	+ 6	— 10	12	149	148	298	+ 149
2989	24·5	— 42 10	6·2	— 1	— 29	— 20	35	235	151	285	+ 50
2998	25·6	— 45 55	6·8	— 3	— 25	— 10	27	248	149	292	+ 44
3001	26·0	— 40 6	7·4	0	— 54	+ 10	55	281	151	281	0
3016	28·9	— 47 11	7·2	— 4	— 35	— 30	46	229	148	292	+ 63
3033	31·7	— 51 23	7·6	— 6	— 40	+ 40	57	315	146	297	— 18
3036	32·1	— 50 32	6·2	— 5	— 12	— 20	23	211	146	296	+ 85
3064	36·3	— 44 45	5·79	— 1	+ 26	0	26	90	148	288	— 162
3069	37·1	— 48 28	6·2	— 4	— 14	+ 10	17	306	147	291	— 15
3070	37·1	— 46 52	4·98	— 3	— 15	— 10	18	236	147	290	+ 54
3073	37·7	— 44 58	5·38	— 1	— 6	— 20	21	197	148	288	+ 91
3075	38·2	— 47 39	5·65	— 3	— 35	— 20	40	240	147	290	+ 50
3089	39·7	— 49 22	5·41	— 4	+ 16	0	16	90	145	289	— 161
3107	43·1	— 45 42	7·1	— 1	— 36	— 20	41	241	146	286	+ 45
3127	45·4	— 44 51	5·11	0	— 16	0	16	270	147	287	+ 17
3133	46·3	— 46 3	5·40	— 1	— 5	— 40	40	187	145	287	+ 100
3140	47·3	— 40 31	6·8	— 3	— 19	— 10	21	242	147	276	+ 34
3156	49·6	— 47 3	5·7	— 1	— 15	— 50	52	197	145	285	+ 88
3166	50·9	— 44 34	6·8	+ 1	— 17	0	17	270	147	281	+ 11
3181	53·8	— 48 5	6·2	— 1	— 14	— 20	24	215	144	286	+ 71
3190	54·6	— 46 45	5·38	0	— 107	+ 30	110	285	144	285	0
3191	55·2	— 47 48	7·7	0	— 24	— 40	47	211	144	286	+ 75
3196	56·7	— 51 4	7·4	— 3	— 30	— 30	42	225	143	290	+ 65
3203	57·8	— 51 42	5·58	— 3	— 20	+ 10	22	297	142	290	— 7
3223	59·8	— 46 36	3·91	+ 1	— 46	— 40	61	229	143	282	+ 53
3243	9 3·2	— 50 43	7·1	— 2	— 2	— 10	10	191	141	286	+ 95
3266	6·3	— 42 45	6·8	+ 4	+ 4	— 60	60	176	143	277	+ 101
3281	8·6	— 41 46	6·8	+ 6	— 41	+ 30	51	306	143	275	— 31
3283	9·2	— 46 50	6·2	+ 2	— 25	— 10	27	248	142	281	+ 33
3296	10·7	— 45 2	6·66	+ 3	— 38	— 30	48	232	142	279	+ 47
3302	11·8	— 43 45	5·11	+ 4	— 28	— 30	41	223	142	277	+ 54
3316	13·9	— 50 32	5·51	0	— 31	+ 10	33	288	141	284	— 4

TABLE I.

Gill — 40° to — 52°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
3339	9 17·8	— 45 31	7·2	+ 4	— 0·005	+ 0·010	0·011	333	141	277	— 56
3352	19·0	— 47 45	7·1	+ 2	+ 6	— 10	12	149	141	281	+ 132
3369	22·2	— 50 38	8·4	+ 1	— 31	0	31	270	139	283	+ 13
3392	25·8	— 50 58	5·78	+ 1	— 12	0	12	270	138	283	+ 13
3402	27·4	— 40 6	5·54	+ 9	+ 15	— 10	18	124	139	270	+ 146
3419	29·8	— 50 42	5·35	+ 1	+ 7	— 20	21	161	137	280	+ 119
3432	31·9	— 48 12	6·8	+ 3	— 24	— 40	47	211	138	279	+ 68
3437	32·3	— 48 48	4·58	+ 3	— 122	+ 10	120	275	138	278	+ 3
3444	33·2	— 42 38	5·58	+ 8	+ 26	— 60	65	157	138	271	+ 114
3479	39·0	— 50 41	6·9	+ 2	— 12	0	12	270	136	278	+ 8
3501	41·7	— 44 11	5·91	+ 8	— 6	— 10	12	211	136	271	+ 60
3516	44·2	— 42 54	7·1	+ 9	— 18	— 10	21	241	137	270	+ 29
3521	45·1	— 45 9	5·47	+ 7	— 16	— 20	26	219	136	272	+ 53
3531	46·5	— 45 37	5·78	+ 7	— 26	+ 10	28	291	135	271	— 20
3533	46·9	— 45 58	4·62	+ 7	— 36	— 40	54	222	135	272	+ 50
3549	49·3	— 50 34	6·2	+ 4	— 2	— 20	20	186	135	276	+ 90
3558	50·2	— 49 39	6·02	+ 4	— 32	0	32	270	135	275	+ 5
3576	52·9	— 47 49	6·2	+ 6	— 25	— 40	47	212	135	274	+ 62
3610	58·4	— 46 2	6·6	+ 8	— 46	— 20	50	246	134	270	+ 24
3623	59·8	— 51 12	7·7	+ 4	— 30	— 20	36	236	133	275	+ 39
3625	10 0·1	— 41 34	6·8	+ 12	+ 26	0	26	90	133	266	+ 176
3626	0·3	— 50 43	6·2	+ 4	— 31	— 10	33	252	133	274	+ 22
3632	1·2	— 46 46	5·37	+ 8	+ 57	— 70	90	141	133	271	+ 130
3649	4·2	— 51 12	5·33	+ 4	+ 7	— 10	12	145	132	274	+ 129
3675	8·6	— 50 37	5·73	+ 5	— 31	— 50	59	212	132	273	+ 61
3677	8·7	— 51 8	5·7	+ 5	— 59	+ 10	60	280	132	274	— 6
3707	13·1	— 41 3	6·2	+ 14	— 19	— 10	21	242	131	263	+ 21
3717	15·2	— 47 4	5·79	+ 9	+ 16	— 30	34	152	131	269	+ 117
3729	16·9	— 41 1	5·05	+ 14	— 41	+ 40	57	314	130	262	— 52
3754	19·9	— 41 50	5·7	+ 13	— 7	— 20	21	199	130	263	+ 64
3795	27·2	— 43 59	6·8	+ 12	— 17	— 40	43	203	128	264	+ 61
3818	30·6	— 48 35	7·6	+ 9	— 23	— 40	46	210	128	267	+ 57
3832	33·3	— 42 6	6·2	+ 15	— 29	— 30	42	224	127	261	+ 37
3879	41·4	— 48 46	2·98	+ 9	+ 75	— 60	96	129	126	265	+ 136
3892	43·2	— 47 5	7·7	+ 11	— 76	+ 20	79	285	126	263	— 22
3912	47·0	— 45 33	8·1	+ 12	— 26	— 30	40	221	125	261	+ 40
3930	49·8	— 42 21	6·5	+ 16	— 40	— 10	41	256	124	259	+ 3
3957	54·3	— 43 8	6·29	+ 15	— 28	— 10	30	250	123	259	+ 9
3958	54·4	— 41 33	4·75	+ 17	+ 26	— 20	33	128	123	258	+ 130
3988	58·9	— 47 0	6·09	+ 12	— 117	+ 10	120	275	123	261	— 14
4002	11 0·1	— 46 46	7·5	+ 13	— 15	— 10	18	236	123	260	+ 24
4011	1·9	— 47 58	7·0	+ 11	— 24	0	24	270	123	261	— 9
4014	2·7	— 51 44	7·0	+ 8	— 39	+ 100	110	338	124	263	— 75
4038	6·4	— 45 35	6·8	+ 14	— 57	0	57	270	122	258	— 12
4042	6·9	— 48 25	5·7	+ 11	— 104	+ 10	100	276	122	260	— 16
4102	16·4	— 43 57	7·2	+ 16	— 71	— 50	87	235	119	256	+ 21
4115	18·4	— 41 59	6·2	+ 18	— 18	— 30	35	211	119	255	+ 44
4125	19·5	— 41 59	7·7	+ 18	— 29	— 10	31	251	119	255	+ 4
4158	24·9	— 41 14	6·8	+ 19	— 64	— 40	75	238	118	253	+ 15
4188	29·2	— 46 41	5·88	+ 14	— 86	— 30	91	251	118	255	+ 4

TABLE I.

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
4192	11 29·9	− 46 57	5·62	+ 14	+0·026	− 0 060	0·065	157	118	256	+ 99
4214	33·8	− 49 47	6·96	+ 11	− 42	− 10	43	256	117	257	+ 1
4252	39·6	− 45 0	5·59	+ 16	− 69	− 10	70	262	116	253	− 9
4281	44·9	− 44 29	4·76	+ 17	− 81	− 10	82	263	115	253	− 10
4282	45·0	− 43 14	6·6	+ 19	− 28	− 30	41	223	115	251	+ 28
4318	50·8	− 51 24	6·8	+ 10	+ 82	− 20	84	104	116	255	+ 151
4331	52 9	− 51 0	6·4	+ 11	− 12	− 10	16	230	115	254	+ 24
4335	53·2	− 45 56	7·4	+ 16	+ 47	− 20	51	113	114	252	+ 139
4385	12 1·8	− 48 0	5·82	+ 14	− 4	− 40	40	186	113	251	+ 65
4388	2·4	− 43 38	6·20	+ 18	− 50	− 70	86	216	112	249	+ 33
4424	8 5	− 40 26	8·0	+ 22	− 350	− 110	370	252	110	247	− 5
4441	11·0	− 48 14	7·7	+ 14	− 274	− 60	280	257	112	249	− 8
4444	11·5	− 51 37	6·8	+ 11	− 2	0	2	270	112	250	− 20
4484	18·5	− 41 49	6·35	+ 21	− 119	− 40	130	252	108	246	− 6
4492	19·8	− 50 45	5·24	+ 12	− 50	− 40	64	231	111	249	+ 18
4496	20·2	− 48 13	6 2	+ 14	− 604	− 90	610	261	110	248	− 13
4502	21·3	− 49 32	4·31	+ 13	− 23	− 40	46	210	111	249	+ 39
4546	29·0	− 40 20	5·65	+ 22	− 122	− 30	120	256	106	244	− 12
4560	30·9	− 47 51	4·16	+ 15	− 206	− 20	210	264	108	246	− 18
4586	35·7	− 48 7	4·80	+ 15	− 94	− 40	102	247	107	245	− 2
4661	48·0	− 43 28	6·2	+ 19	− 225	− 250	330	222	104	242	+ 20
4666	48·4	− 42 14	5·57	+ 20	− 41	− 50	65	219	103	242	+ 23
4676	49·9	− 50 31	5 38	+ 12	− 21	− 30	37	215	107	244	+ 29
4721	57·8	− 40 31	6·8	+ 22	− 31	− 40	51	218	101	241	+ 23
4734	59·5	− 40 55	6·0	+ 21	+ 3	− 40	40	176	101	240	+ 64
4763	13 4·0	− 41 34	5·95	+ 21	− 86	− 40	95	245	100	239	− 6
4765	4·3	− 42 42	5·25	+ 20	− 118	− 60	130	243	101	239	− 4
4778	6·0	− 50 2	6·16	+ 12	− 41	− 30	51	234	104	241	+ 7
4812	11·1	− 50 37	6·2	+ 12	− 21	− 30	37	215	103	240	+ 25
4854	18·8	− 48 8	7·2	+ 14	− 24	− 40	47	211	101	239	+ 28
4858	19·6	− 40 51	5·89	+ 21	− 8	− 40	41	191	98	237	+ 46
4868	21·8	− 50 31	5·51	+ 11	− 2	− 20	20	186	102	239	+ 53
4911	29·1	− 41 46	6·8	+ 20	− 30	− 10	32	252	96	236	− 16
4913	29·6	− 43 30	6·2	+ 18	− 61	− 40	73	237	97	236	− 1
4916	29·8	− 45 47	6·0	+ 16	+ 5	− 40	40	173	98	236	+ 63
4934	32·3	− 49 19	6·5	+ 12	− 111	0	110	270	101	237	− 33
4959	36·2	− 41 26	6·2	+ 20	− 30	− 10	32	252	95	235	− 17
4972	38·7	− 50 48	4·78	+ 11	+ 7	− 40	41	170	100	236	+ 66
5003	44·0	− 46 17	6·02	+ 15	− 5	− 50	50	186	97	234	+ 48
5019	46·2	− 46 31	6·8	+ 15	− 46	− 20	50	246	96	233	− 13
5044	51·0	− 44 12	4·22	+ 17	− 17	− 30	34	210	95	233	+ 23
5065	53·4	− 43 36	6·8	+ 17	− 105	− 70	120	236	94	232	− 4
5066	53·9	− 45 0	4·67	+ 16	− 6	− 40	40	189	95	232	+ 43
5085	58·4	− 40 35	4·76	+ 20	− 20	− 40	45	207	91	231	+ 24
5094	59·6	− 48 7	6·8	+ 13	− 54	− 40	67	233	96	231	− 2
5124	14 5·2	− 41 3	6·8	+ 19	− 41	− 20	46	244	90	230	− 14
5170	12·7	− 44 37	4·98	+ 15	+ 15	− 110	110	172	91	229	+ 57
5208	18·1	− 44 49	4·52	+ 15	− 17	− 20	26	220	91	228	+ 8
5220	19·8	− 43 46	7·4	+ 15	− 39	− 70	80	209	90	228	+ 19
5249	24·2	− 49 54	4·78	+ 9	− 42	− 20	46	244	94	227	− 17

TABLE I

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ'_α	μ_δ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
5259	14 26·2	− 41 33	6.8	+ 17	−0·019	−0·030	0 036	212	88	227	+ 15
5268	27·6	− 40 58	6·2	+ 17	− 8	− 30	31	195	88	226	+ 31
5278	29·5	− 48 53	4·21	+ 10	− 23	− 40	46	210	93	226	+ 16
5279	29·6	− 42 34	7·4	+ 16	− 29	− 30	42	224	88	226	+ 2
5288	31·0	− 46 2	6·8	+ 13	− 202	− 250	320	219	91	226	+ 7
5332	38·1	− 46 55	6·8	+ 11	− 45	− 40	60	228	91	225	− 3
5336	39·4	− 43 2	6·8	+ 15	− 18	− 20	27	222	87	223	+ 1
5362	42·9	− 42 18	6·8	+ 15	− 52	− 30	60	240	86	224	− 16
5440	56·6	− 46 34	4·01	+ 10	− 15	− 40	43	201	88	221	+ 20
5470	15 0·5	− 44 48	4·51	+ 11	− 17	− 30	34	210	86	220	+ 10
5490	4·5	− 44 1	5·14	+ 11	− 28	− 40	49	215	85	219	+ 4
5504	7·2	− 47 36	7·2	+ 8	+ 16	− 50	52	162	87	219	+ 57
5509	7·9	− 41 1	5·36	+ 13	+ 4	− 10	11	158	82	220	+ 62
5510	7·9	− 43 1	6·8	+ 12	− 62	− 30	69	244	84	219	− 25
5529	10·8	− 40 20	5·86	+ 14	− 20	− 30	36	214	81	219	+ 5
5545	13·3	− 47 51	6·8	+ 7	− 1 640	− 270	1·700	259	87	218	− 41
5547	13·4	− 47 28	5·15	+ 8	− 115	− 140	180	218	87	218	0
5618	25·1	− 40 4	6·8	+ 13	+ 37	− 70	79	152	79	217	+ 65
5631	27·3	− 44 32	4·93	+ 9	− 6	− 50	50	187	83	214	+ 27
5647	29·7	− 42 9	4·21	+ 10	− 152	+ 50	160	288	81	216	− 72
5666	32·6	− 44 15	4·68	+ 8	− 189	− 260	320	216	83	215	− 1
5723	41·5	− 45 1	6·35	+ 7	+ 58	− 40	70	125	82	212	+ 87
5735	43·6	− 50 14	6·47	+ 3	+ 16	0	16	90	86	211	+ 121
5772	49·7	− 43 43	6·8	+ 7	− 62	− 60	86	226	80	211	− 15
5779	51·0	− 41 23	5·04	+ 9	− 41	− 20	46	244	78	211	− 33
5816	57·7	− 44 50	4·95	+ 5	+ 15	+ 20	25	37	81	209	+ 172
5822	58·3	− 48 5	6·8	+ 2	+ 16	− 40	43	158	83	208	+ 50
5846	16 2·8	− 40 47	6·2	+ 7	− 110	− 140	180	218	76	209	− 9
5853	3·8	− 49 34	7·2	+ 1	− 43	− 20	47	245	83	207	− 38
5888	10·6	− 42 21	5·81	+ 5	+ 4	− 20	20	169	77	207	+ 38
5899	12·2	− 47 53	6.79	+ 1	− 4	− 30	30	188	82	206	+ 18
5905	13·1	− 49 16	5·57	0	+ 6	− 30	31	169	83	206	+ 37
5907	13·7	− 43 36	6·2	+ 4	− 61	− 30	68	244	77	207	− 37
5981	25·3	− 49 8	7·1	− 1	− 14	− 20	24	215	82	204	− 11
5996	27·6	− 42 36	5·70	+ 3	− 7	− 30	31	193	76	205	+ 12
6014	31·3	− 48 30	7·4	− 1	+ 6	− 10	12	149	81	203	+ 54
6044	36·1	− 40 36	5·85	+ 3	− 20	− 60	63	198	73	201	+ 3
6087	43·8	− 40 30	6·8	+ 2	− 20	− 40	45	207	72	200	− 7
6090	44·0	− 41 34	8·1	+ 2	− 19	− 20	28	224	73	200	− 24
6098	45·2	− 42 9	5·13	+ 1	+ 37	− 30	48	129	74	199	+ 70
6108	46·5	− 50 28	6·5	− 4	− 2	− 30	30	184	82	198	+ 14
6109	46·6	− 43 7	6·8	0	− 28	− 20	34	234	79	198	− 36
6116	47·9	− 40 37	6·8	+ 1	+ 3	− 20	20	171	72	199	+ 28
6169	56·8	− 43 56	6·2	− 2	+ 15	+ 10	18	56	76	197	+ 141
6190	17 0·5	− 46 52	7·4	− 4	− 15	+ 10	18	304	78	196	− 108
6193	1·7	− 44 24	5·10	− 3	− 27	− 50	57	208	75	196	− 12
6271	15·2	− 44 2	5·7	− 5	− 28	− 30	41	223	75	194	− 29
6287	17·0	− 51 50	6·8	− 9	− 11	− 30	32	200	83	192	− 8
6290	17·5	− 50 31	5.6	− 9	− 12	− 30	32	202	80	192	− 10
6318	22 6	− 45 57	7·1	− 7	− 67	− 20	70	253	77	190	− 63

TABLE I.

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
6328	17 24·8	− 48 26	6·8	− 9	− 0·024	+ 0·050	0·055	335	78	190	− 145
6338	26·3	− 46 25	4·71	− 8	− 25	− 40	47	212	77	190	− 22
6365	30·8	− 49 20	5·02	− 10	+ 114	− 180	210	148	80	189	+ 41
6384	34·4	− 45 54	7·1	− 9	− 5	− 10	11	207	76	187	− 20
6428	41·4	− 40 3	5·06	− 7	+ 3	− 30	30	174	70	187	+ 13
6431	42·5	− 45 33	6·8	− 9	− 16	− 10	19	238	75	187	− 51
6459	47·6	− 44 19	5·04	− 9	− 38	− 20	43	242	74	186	− 56
6469	48·9	− 41 42	4·88	− 9	− 19	− 20	28	224	72	185	− 39
6510	56·9	− 50 6	4·01	− 14	− 3	− 30	30	186	79	184	− 2
6514	57·8	− 43 26	5·34	− 11	+ 15	− 120	120	173	73	183	+ 10
6535	18 1·6	− 47 32	6·2	− 13	− 14	− 40	42	199	77	183	− 16
6546	3·2	− 50 35	7·2	− 15	− 21	− 80	83	195	80	182	− 13
6568	6·8	− 44 15	5·54	− 13	+ 58	+ 20	61	71	74	182	+ 111
6572	7·2	− 51 6	6·2	− 16	− 21	− 50	54	203	81	182	− 21
6585	9·7	− 42 20	6·5	− 13	+ 4	− 30	30	172	72	182	+ 10
6619	15·2	− 44 10	5·62	− 14	+ 15	− 30	34	153	74	180	+ 27
6634	17·7	− 46 2	3·86	− 15	− 5	− 50	50	186	76	179	− 7
6646	19·2	− 49 8	4·17	− 17	+ 155	− 260	300	149	79	178	+ 29
6661	22·5	− 46 0	5·23	− 16	− 5	− 50	50	186	76	178	− 8
6703	29·8	− 48 1	6·2	− 18	+ 26	+ 20	33	52	78	176	+ 124
6745	37·3	− 50 13	6·66	− 20	+ 36	− 40	54	138	80	176	+ 38
6748	37·8	− 49 45	7·7	− 20	− 32	− 10	33	253	80	175	− 78
6754	39·0	− 40 32	5·30	− 17	− 20	− 30	36	214	70	172	− 42
6758	39·8	− 43 49	5·86	− 18	+ 26	− 20	33	128	73	174	+ 46
6774	43·1	− 46 44	5·61	− 20	+ 26	− 20	33	128	77	173	+ 45
6776	43·4	− 41 51	6·8	− 18	− 7	− 30	31	193	72	173	− 20
6794	47·0	− 49 9	7·0	− 21	− 33	− 50	60	213	79	172	− 41
6821	51·2	− 48 27	7·7	− 21	− 24	− 40	47	211	78	172	− 39
6845	55·2	− 48 35	7·0	− 22	+ 36	− 40	54	138	78	170	+ 32
6856	57·3	− 50 30	6·2	− 22	− 31	− 40	51	218	80	170	− 48
6901	19 5·5	− 45 24	6·05	− 23	+ 89	− 70	113	128	76	168	+ 40
6909	7·2	− 45 41	5·45	− 23	− 5	− 10	11	207	76	168	− 39
6943	12·9	− 51 28	6·2	− 26	+ 64	+ 10	65	81	82	167	+ 86
6956	15·2	− 40 51	4·30	− 23	+ 26	− 140	140	170	72	166	− 4
7006	24·3	− 45 32	5·89	− 26	− 16	− 40	43	202	77	163	− 39
7015	25·9	− 48 22	5·12	− 27	− 14	− 30	33	205	80	165	− 40
7100	41·1	− 47 52	6·8	− 29	− 14	− 40	42	199	80	160	− 39
7103	42·0	− 42 11	8·0	− 29	+ 37	− 30	48	129	74	159	+ 30
7115	43·4	− 40 12	5·55	− 29	+ 49	− 30	58	122	72	160	+ 38
7131	46·6	− 42 12	4·25	− 29	+ 4	+ 40	40	6	74	158	+ 152
7162	51·7	− 43 23	6·2	− 30	+ 15	0	15	90	76	159	+ 69
7163	51·9	− 45 27	5·99	− 31	+ 5	− 30	30	171	78	159	− 12
7219	20 1·4	− 44 15	7·0	− 32	+ 4	0	4	90	78	156	+ 66
7237	3·8	− 43 8	6·0	− 33	− 18	− 120	120	189	77	155	− 34
7272	10·2	− 44 55	8·0	− 34	+ 47	+ 30	56	57	79	154	+ 97
7282	12·6	− 50 23	5·9	− 35	− 335	− 250	410	233	84	154	− 79
7292	14·0	− 42 26	5·90	− 35	+ 59	− 110	130	151	77	154	+ 3
7297	15·4	− 42 49	5·90	− 35	− 18	0	18	270	77	153	− 117
7357	25·3	− 44 56	5·29	− 37	+ 15	− 40	43	159	80	152	− 7
7377	28·8	− 47 43	3·37	− 38	+ 56	+ 40	69	54	83	150	+ 96

Gill — 40° to — 52°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
7393	20 31·6	− 47 16	8·2	− 38	− 0·005	− 0·020	0·021	194	83	150	− 44
7404	33·7	− 45 20	7·13	− 39	− 6	+ 10	12	329	81	149	− 180
7437	38·8	− 46 19	7·4	− 39	− 15	− 10	18	236	82	149	− 87
7452	40·9	− 46 41	5·07	− 39	+ 47	+ 10	48	78	83	148	+ 70
7461	42·4	− 52 4	5·27	− 39	+ 17	− 20	26	140	88	148	+ 8
7478	45·5	− 40 17	5·49	− 40	+ 60	− 100	120	149	77	147	− 2
7500	49·4	− 44 35	6·2	− 42	− 552	− 980	1 129	209	81	146	− 63
7508	51·4	− 51 45	6·01	− 41	− 87	+ 130	160	325	89	147	− 178
7509	51·7	− 43 30	7·1	− 41	+ 15	+ 10	18	56	81	146	+ 90
7544	58·3	− 41 53	5·72	− 43	+ 26	− 20	33	128	81	144	+ 16
7550	59·2	− 49 26	6·2	− 43	+ 26	− 50	56	153	87	144	− 9
7551	59·4	− 45 53	5·7	− 43	+ 26	− 40	48	147	84	144	− 3
7563	21 1·9	− 43 53	6·8	− 43	− 7	− 40	41	190	83	144	− 46
7602	9·4	− 49 14	6·8	− 44	− 33	− 90	96	200	88	143	− 57
7614	11·0	− 47 34	6·2	− 44	+ 26	− 50	56	153	87	142	− 11
7638	16·0	− 47 9	5·7	− 45	+ 67	− 40	78	121	87	141	+ 20
7645	16·4	− 41 32	6·04	− 46	+ 15	+ 10	18	56	83	141	+ 85
7662	19·0	− 43 5	5·73	− 46	− 61	0	61	270	84	141	− 129
7690	24·2	− 41 44	5·53	− 48	− 8	+ 10	13	321	84	140	+ 179
7752	35·1	− 44 4	6·8	− 49	+ 210	− 80	230	111	87	137	+ 26
7770	38·1	− 47 58	8·0	− 49	− 14	0	14	270	90	137	− 133
7783	40·2	− 47 52	5·86	− 50	+ 146	− 320	350	155	90	136	− 19
7785	40·5	− 48 21	6·8	− 49	− 14	− 10	17	234	90	136	− 98
7791	41·7	− 47 11	7·7	− 50	+ 26	− 60	65	157	90	136	− 21
7930	22 7·1	− 41 59	6.2	− 55	+ 559	− 800	980	145	89	132	− 13
7981	15·4	− 46 33	5·98	− 56	+ 232	− 70	240	107	93	131	+ 24
8017	22·3	− 44 23	4·43	− 57	− 38	0	38	270	93	129	− 141
8037	24·8	− 43 54	7·4	− 58	− 6	+ 10	12	329	94	129	+ 160
8057	29·1	− 48 57	6·8	− 57	+ 46	− 30	55	123	97	128	+ 5
8059	29·2	− 41 14	6·29	− 60	+ 15	− 80	81	169	92	128	− 41
8066	30·6	− 47 21	7·4	− 58	+ 127	− 70	150	119	96	128	+ 9
8075	32·5	− 50 15	7·47	− 57	+ 64	− 70	95	138	98	128	− 10
8090	35·2	− 47 51	6·2	− 58	+ 6	− 340	350	170	98	127	− 43
8091	35·2	− 47 33	2·26	− 58	+ 167	− 30	170	100	97	127	+ 27
8093	35·4	− 44 54	6·43	− 59	+ 37	+ 30	48	51	95	127	+ 76
8097	36·3	− 42 4	5·00	− 60	+ 4	− 100	100	177	94	127	− 50
8108	38·3	− 47 12	5·50	− 59	− 45	− 20	49	246	97	127	− 119
8112	38·6	− 49 38	6·2	− 58	+ 210	− 50	220	104	98	127	+ 23
8117	39·0	− 50 20	6·67	− 57	+ 111	− 270	290	158	99	126	− 32
8126	41·0	− 51 59	3·86	− 57	+ 128	− 60	140	115	100	126	+ 11
8157	46·2	− 49 16	5·7	− 59	+ 232	− 90	250	112	99	125	+ 13
8214	57·0	− 42 9	5·97	− 64	− 41	+ 60	73	326	97	124	+ 158
8233	59·8	− 44 12	4·57	− 63	− 28	− 30	41	223	99	123	− 100
8234	59·9	− 50 17	6·59	− 60	+ 36	− 10	37	106	102	122	+ 16
8246	23 3·0	− 43 32	6·06	− 65	− 355	− 30	350	265	99	122	− 143
8247	3·2	− 41 16	6·19	− 66	+ 26	− 60	65	157	97	123	− 34
8249	3·3	− 45 55	4·23	− 63	+ 161	− 50	170	107	100	122	+ 15
8274	8·1	− 41 47	5·90	− 67	+ 93	− 120	150	143	98	122	− 21
8306	13·8	− 51 0	6·15	− 61	+ 35	− 90	97	159	104	120	− 39
8321	16·5	− 42 17	6·9	− 67	− 41	− 10	42	256	100	121	− 135

Gill — 40° to — 52°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
8323	23 16·9	— 43 49	6·8	— 66	+ 0·004	— 0·030	0·030	172	101	120	— 52
8345	20·3	— 50 51	6·42	— 62	+ 17	0	17	90	105	119	+ 29
8351	22·2	— 45 11	6·73	— 66	+ 15	— 40	43	159	102	119	— 40
8363	23·8	— 42 41	6·5	— 69	— 84	— 100	130	220	101	120	— 100
8366	24·7	— 45 32	6·00	— 67	+ 26	— 20	33	128	102	119	— 9
8387	28·2	— 43 19	5·04	— 68	+ 37	— 30	48	129	103	118	— 11
8427	35·3	— 42 58	7·0	— 69	+ 15	— 60	62	166	104	117	— 49
8438	37·3	— 45 47	5·9	— 67	+ 309	+ 20	310	86	105	117	+ 31
8451	39·4	— 40 53	6·45	— 71	+ 93	— 40	101	113	103	118	+ 5
8458	40·6	— 50 55	5·52	— 64	+ 7	— 20	21	161	108	115	— 46
8479	44·0	— 48 5	6·8	— 66	— 14	+ 20	24	325	106	116	+ 151
8495	47.1	— 50 8	7·8	— 65	+ 93	+ 10	93	84	109	114	+ 30
8542	54·6	— 49 30	5·75	— 66	— 3	0	3	270	109	113	— 157
8544	54·9	— 51 2	5·45	— 65	— 30	+ 10	32	288	110	113	— 175
8546	55·3	— 40 51	6·8	— 73	+ 37	— 30	48	129	106	116	— 13

Auwers — 52° to — 90°.

4	0 13·6	— 65 36	4·45	— 51	+ 1·675	+ 1·147	2·000	56	117	102	+ 46
5	19·5	— 77 58	3·06	— 39	+ 2·202	+ 311	2·200	82	119	95	+ 13
10	28·5	— 53 4	5·61	— 64	+ 183	+ 15	180	86	116	106	+ 20
13	37·7	— 58 9	4·66	— 59	— 15	— 5	16	252	118	102	— 150
16	44·2	— 75 37	5·12	— 42	+ 97	+ 3	97	88	120	91	+ 3
17	50·4	— 70 12	5·54	— 47	— 50	— 45	67	228	123	92	— 136
19	56·8	— 57 41	6·03	— 60	— 62	+ 5	62	275	120	98	— 177
21	1 2·3	— 62 27	5·40	— 55	+ 50	— 5	50	96	121	95	— 1
24	20·7	— 65 1	5·93	— 52	— 18	— 15	23	230	123	89	— 141
28	32·9	— 79 8	6·16	— 38	— 40	— 145	150	196	122	76	— 120
I	33·1	— 57 52	0.68	— 59	+ 74	— 35	82	115	125	89	— 26
31	41·3	— 79 47	6·23	— 37	+ 38	+ 15	41	68	123	74	+ 6
32	41·4	— 54 9	5·24	— 61	+ 81	+ 52	96	57	127	92	+ 35
34	51·2	— 52 14	3·79	— 62	+ 618	+ 261	670	67	127	91	+ 24
35	51·8	— 68 16	4·82	— 48	+ 64	+ 57	86	48	126	80	+ 32
37	54·9	— 62 11	3·13	— 53	+ 240	+ 15	240	87	127	82	— 5
43	2 19·5	— 69 14	4·41	— 46	— 65	+ 11	66	280	128	72	+ 152
44	21·4	— 60 52	5·52	— 52	— 71	— 136	160	207	130	78	— 129
48	34·4	— 79 39	5·44	— 37	+ 102	— 34	100	109	125	60	— 49
49	33·3	— 53 5	5·28	— 57	+ 20	— 45	49	156	135	81	— 75
51	37·6	— 68 48	4·34	— 45	+ 79	+ 2	79	89	129	67	— 22
54	56·5	— 64 34	5·15	— 47	— 1	— 36	36	182	133	65	— 117
57	3 0·7	— 60 14	5·23	— 50	— 88	— 55	104	238	136	68	— 170
58	2·0	— 72 24	5·53	— 41	+ 13	+ 1	13	86	130	57	— 29
60	9·4	— 57 47	5·88	— 50	— 19	— 28	28	224	137	68	— 156
63	19·0	— 77 51	5·70	— 36	+ 102	+ 63	120	57	128	49	— 8
64	27·6	— 63 22	4·97	— 45	+ 320	+ 353	480	42	137	62	+ 20
70	42·6	— 65 12	3·93	— 43	+ 289	+ 60	300	78	137	51	— 27
72	49·2	— 74 37	3·29	— 37	+ 37	+ 105	100	16	131	42	+ 26
73	51·3	— 53 3	6·46	— 47	— 1	— 57	57	181	145	62	— 119

Auwers — 52° to — 90°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
75	3 56·7	— 61 45	4·48	— 43	— 0·021	— 0·021	0·030	225	140	50	— 175
78	4 12·8	— 62 47	3·52	— 41	+ 22	+ 49	54	24	141	44	+ 20
82	20·6	— 63 41	5·35	— 40	+ 70	+ 145	160	26	141	41	+ 15
83	26·5	— 80 30	5·79	— 32	+ 11	+ 88	89	7	127	30	+ 23
86	31·3	— 55 18	3·70	— 40	+ 34	— 8	35	103	148	46	— 57
89	42·5	— 59 58	5·58	— 38	— 9	+ 42	43	348	145	36	+ 48
90	44·3	— 71 9	5·92	— 35	— 7	+ 34	35	348	136	29	+ 41
93	5 3·4	— 57 39	4·87	— 36	— 74	+ 110	130	326	149	30	+ 64
94	4·3	— 71 29	5·42	— 34	— 22	+ 41	47	332	137	21	+ 49
95	13·2	— 82 38	6·03	— 30	— 29	+ 1	29	272	126	17	+ 105
96	13·8	— 67 19	4·92	— 33	— 7	+ 37	38	349	141	21	+ 32
103	32·6	— 62 34	3·87	— 32	— 17	+ 17	24	315	146	17	+ 62
104	36·8	— 76 26	5.14	— 30	+ 83	+ 323	330	14	133	11	— 3
107	44·6	— 65 47	4·66	— 31	— 53	— 14	55	255	143	11	+ 116
110	47·6	— 56 12	4·46	— 29	+ 46	— 70	84	147	153	11	— 136
112	58·7	— 79 23	5·70	— 29	— 39	+ 97	105	338	130	4	+ 26
115	6 5·9	— 62 8	5·14	— 28	— 2	— 80	80	181	147	2	— 179
116	7·8	— 54 57	6·01	— 27	— 47	— 20	51	247	154	2	+ 115
IV	21·2	— 52 38	−0·79	— 24	+ 11	+ 7	13	58	156	355	— 63
121	26·5	— 69 37	5·61	— 27	— 14	+ 209	210	357	140	356	— 1
123	32·3	— 52 53	4·63	— 23	— 37	— 14	40	249	156	349	+ 100
126	46·9	— 61 49	3·46	— 23	— 89	+ 277	290	342	147	346	+ 4
128	50·4	— 80 41	5·79	— 26	— 19	+ 75	77	346	128	350	+ 4
129	52·9	— 70 48	5·70	— 25	— 9	+ 9	13	315	138	346	+ 31
132	7 2·0	— 56 34	5·48	— 19	— 24	— 21	40	238	151	337	+ 99
133	4·4	— 79 14	5·63	— 26	— 26	— 22	34	230	130	345	+ 115
138	16·9	— 67 43	4·09	— 22	— 8	— 16	18	207	140	338	+ 131
141	32·6	— 52 15	5·08	— 14	— 1	— 32	32	182	153	321	+ 139
145	43·4	— 72 18	3·99	— 21	+ 11	— 2	11	100	136	331	— 129
149	48·9	— 65 52	6·17	— 18	— 27	— 12	30	246	140	326	+ 80
151	53·6	— 52 39	3·73	— 12	— 46	+ 14	48	287	150	310	+ 23
156	8 20·0	— 59 6	1·91	— 12	— 33	+ 2	33	274	143	310	+ 36
157	24·3	— 77 5	4·36	— 21	— 149	+ 17	150	278	130	322	+ 44
158	24·3	— 65 43	3·78	— 15	— 50	— 184	190	196	138	314	+ 118
162	37·9	— 59 19	4·57	— 10	— 41	— 30	51	234	142	304	+ 70
164	41·3	— 54 15	2·17	— 7	+ 4	— 97	97	178	143	297	+ 119
166	45·5	— 78 30	5·85	— 21	— 54	+ 15	56	286	129	317	+ 31
168	52·2	— 60 10	4·12	— 9	— 23	+ 46	51	334	139	300	— 34
170	9 0·5	— 65 54	4·30	— 12	— 26	— 112	110	195	135	304	+ 109
172	4·8	— 72 6	4·70	— 16	— 71	+ 3	71	272	131	307	+ 35
173	11·8	— 69 12	1·90	— 14	— 170	+ 90	190	298	133	303	+ 5
174	13·7	— 58 45	2·41	— 6	— 41	— 4	41	264	137	294	+ 30
176	18·3	— 54 29	2·76	— 2	— 40	+ 11	42	285	138	288	+ 3
VI	27·5	— 56 29	3·15	— 3	— 53	+ 6	53	277	136	286	+ 9
180	30·6	— 72 32	5·65	— 15	— 42	— 4	42	264	130	300	+ 36
181	30·8	— 58 41	4·35	— 4	— 25	+ 6	26	284	136	290	+ 6
184	41·8	— 61 57	3·7	— 6	— 31	+ 3	31	276	133	289	+ 13
VII	44·0	— 64 29	3·16	— 8	— 26	+ 7	27	285	133	290	+ 5
186	52·4	— 53 59	3·84	+ 1	— 32	— 5	32	261	133	279	+ 18
190	10 10·7	— 69 25	3·78	— 10	— 22	+ 2	22	275	128	287	+ 12

TABLE I.

Auwers — 52° to — 90°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ — ψ
	h m	° ′		°	″	″	″	°	°	°	°
193	10 21·9	— 73 25	4·19	— 13	— 0·040	— 0·027	0·048	236	126	288	+ 52
195	23·3	— 58 6	4·22	0	— 11	— 4	12	250	129	275	+ 25
196	27·6	— 61 3	3·79	— 3	— 29	— 3	29	264	128	277	+ 13
199	34·0	— 77 57	4·26	— 17	— 57	+ 27	63	296	123	287	— 9
200	34·3	— 54 57	4·57	+ 3	— 56	— 6	56	264	127	272	+ 8
201	38·5	— 63 44	3·18	— 4	— 34	— 4	34	263	127	278	+ 15
VIII	40·3	— 59 2	1·	0	— 11	+ 13	17	320	126	272	— 48
203	44·6	— 79 52	4·79	— 19	— 53	— 5	53	265	123	286	+ 21
204	48·4	— 58 11	4·02	+ 1	+ 55	+ 24	60	66	126	271	— 155
209	11 3·2	— 58 18	4·20	+ 2	— 30	+ 5	30	280	123	267	— 13
212	15·3	— 53 48	4·55	+ 6	— 46	— 15	48	252	122	262	+ 10
216	30·0	— 62 19	3·48	0	— 59	— 24	64	248	121	263	+ 15
217	32·1	— 75 12	5·83	— 13	— 146	— 15	150	266	121	272	+ 6
219	39·7	— 66 2	3·94	— 4	— 121	+ 25	120	282	120	263	— 19
220	40·5	— 60 29	4·45	+ 2	— 30	— 31	43	224	119	261	+ 37
223	51·9	— 55 37	5·75	+ 6	— 48	— 31	57	237	117	257	+ 20
224	53·4	— 77 31	5·23	— 15	— 52	— 19	56	250	120	267	+ 17
225	56·7	— 62 37	4·64	— 1	— 157	— 25	160	261	118	259	— 2
229	12 8·5	— 58 3	3·25	+ 4	— 52	— 12	53	257	116	254	— 3
230	10·8	— 67 16	4·32	— 5	— 253	— 35	260	262	117	257	— 5
231	11·1	— 78 37	4·51	— 16	— 52	+ 14	54	285	119	264	— 21
232	14·6	— 59 42	3·70	+ 3	— 191	+ 80	210	293	115	254	— 39
IX	19·7	— 62 24	1·19	0	— 37	— 26	45	235	115	254	+ 19
237	24·2	— 56 25	1·72	+ 7	+ 18	— 271	270	176	113	251	+ 75
238	25·1	— 71 26	4·18	— 8	— 75	— 11	76	262	117	257	— 5
X	29·8	— 68 27	3·08	— 5	— 43	— 35	55	231	116	254	+ 23
242	38·6	— 67 25	3·43	— 4	— 53	— 19	56	250	114	252	+ 2
243	40·4	— 59 0	1·66	+ 4	— 54	— 30	62	241	111	248	+ 7
246	53·7	— 70 53	3·77	— 8	+ 245	— 31	250	97	115	250	+ 153
248	13 0·3	— 52 47	6·06	+ 9	— 68	— 35	77	243	106	243	0
249	4·5	— 59 15	4·91	+ 4	— 73	— 35	81	244	109	243	— 1
250	6·8	— 67 14	5·08	— 4	— 20	— 25	32	219	112	246	+ 27
257	28·5	— 75 3	6·72	— 13	— 32	— 5	32	261	113	243	— 18
258	31·9	— 72 50	2·75	+ 9	— 40	— 36	54	228	103	237	+ 9
263	48·6	— 63 4	4·93	— 1	— 56	— 85	102	213	106	236	+ 23
XI	55·0	— 59 46	1·60	+ 1	— 42	— 54	68	218	104	234	+ 16
268	14 2·7	— 80 25	5·21	— 18	— 34	— 45	57	217	115	238	+ 21
270	11·6	— 55 49	4·53	+ 5	— 46	— 44	64	226	100	230	+ 4
271	14·7	— 67 38	6·03	— 7	— 41	— 44	60	223	107	232	+ 9
XII	30·9	— 60 19	0·37	0	— 3·604	— 737	3·700	259	101	227	— 32
XIII	32·4	— 64 26	3·54	— 4	— 221	— 245	330	222	103	226	+ 4
277	32·4	— 78 31	3·97	— 17	— 26	— 37	45	215	112	232	+ 17
278	35·4	— 62 21	5·32	— 2	+ 47	— 104	110	153	101	226	+ 73
281	43·8	— 76 9	5·54	— 15	— 66	— 4	66	266	110	228	— 38
282	45·9	— 59 36	5·50	0	— 169	— 134	210	232	98	223	— 9
288	15 7·2	— 68 13	3·17	— 9	— 70	— 42	82	239	103	220	— 19
289	7·7	— 58 20	4·17	— 1	— 118	— 162	200	217	96	220	+ 3
292	17·9	— 72 57	5·78	— 14	— 8	— 43	44	191	106	212	+ 21
293	25·3	— 65 54	4·24	— 8	+ 14	— 88	89	171	101	216	+ 45
296	29·6	— 51 58	5·57	+ 2	— 49	— 63	80	218	89	214	— 4

Auwers — 52° to — 90°.

No.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ — ψ
	h m	° ′		°	″	″	″	°	°	°	°
298	15 44·1	— 63 3	3·26	— 7	— 0·197	— 0·407	0·460	206	97	212	+ 6
304	16 3·6	— 54 18	5·27	— 3	— 45	— 53	70	220	88	207	— 13
305	4·1	— 63 22	4·21	— 9	— 11	— 18	22	210	96	207	— 3
309	15·0	— 69 48	5·16	— 14	+ 175	+ 108	200	58	102	206	+ 148
310	14·3	— 78 36	4·11	— 20	— 131	— 79	150	238	110	208	— 30
311	19·7	— 61 21	5·28	— 9	— 51	— 26	57	243	94	204	— 39
315	25·2	— 77 15	4·39	— 19	— 310	— 342	460	222	108	204	— 18
317	35·4	— 68 48	2·06	— 15	+ 12	— 49	50	166	100	201	+ 35
318	38·9	— 58 49	3·77	— 9	+ 9	— 43	44	168	91	201	+ 33
322	48·2	— 55 47	3·19	— 8	— 48	— 46	67	226	87	199	— 27
324	49·6	— 52 58	4·34	— 7	— 26	+ 3	26	277	85	198	— 79
328	17 8·2	— 70 0	5·78	— 18	— 24	— 41	47	210	100	194	— 16
XVII	14·9	— 56 16	3·59	— 11	— 19	— 11	22	240	86	193	— 47
331	14·9	— 55 25	2·89	— 11	— 16	— 23	28	215	85	193	— 22
334	19·9	— 60 35	3·93	— 14	— 69	— 111	130	212	91	192	— 20
338	27·8	— 54 25	5·39	— 12	— 97	— 161	190	212	84	189	— 23
341	33·4	— 64 40	3·68	— 18	— 28	— 46	54	211	95	188	— 23
347	56·5	— 63 40	4·56	— 20	+ 7	— 220	220	178	94	184	+ 6
351	18 6·6	— 56 4	5·65	— 18	— 60	— 20	63	252	86	182	— 70
353	11·7	— 61 33	4·32	— 20	— 31	+ 10	33	288	91	180	— 108
357	16·9	— 74 2	5·94	— 24	— 2	— 115	110	181	104	179	— 2
360	28·4	— 71 32	4·21	— 25	— 30	— 169	170	190	101	176	— 14
363	34·3	— 73 7	6·37	— 25	— 32	+ 11	34	289	103	174	— 115
365	40·6	— 62 20	4·52	— 24	— 41	— 54	68	217	92	174	— 43
367	44·0	— 67 23	4·0	— 25	— 50	+ 14	52	286	97	174	— 112
369	48·5	— 53 6	5·03	— 22	— 11	+ 11	16	315	83	172	— 143
372	56·4	— 52 31	5·21	— 23	— 20	— 114	110	189	83	170	— 19
376	19 4·6	— 66 52	5·80	— 27	— 17	+ 11	20	303	97	168	— 135
379	17·8	— 54 35	5·60	— 27	— 72	+ 22	75	287	85	166	— 121
383	35·0	— 72 48	5·66	— 30	— 24	+ 34	42	325	104	162	— 163
385	37·8	— 56 40	5·77	— 30	+ 38	— 148	160	165	88	162	— 3
387	43·8	— 61 30	6·34	— 31	— 8	+ 22	23	340	92	161	— 179
389	46·1	— 73 14	4·21	— 31	+ 65	— 132	150	153	104	159	+ 6
393	56·5	— 66 30	3·73	— 32	+ 1·162	— 1·144	1·600	133	97	158	+ 25
394	57·7	— 53 14	4·99	— 33	— 65	+ 13	66	281	85	158	— 123
397	20 15·7	— 57 8	2·21	— 35	— 5	— 77	77	184	90	154	— 30
399	27·1	— 61 57	5·19	— 36	+ 13	— 37	39	161	96	151	— 10
400	26·6	— 76 37	6·29	— 33	+ 191	— 23	190	97	109	150	+ 53
403	33·7	— 66 39	3·71	— 36	— 53	— 10	54	259	100	149	— 110
404	34·9	— 52 22	4·90	— 38	+ 144	— 69	170	116	88	149	+ 33
408	45·0	— 58 55	3·83	— 39	— 7	— 8	11	221	94	148	— 73
410	49·5	— 77 30	5·48	— 34	— 20	— 366	370	183	109	144	— 39
414	21 1·5	— 70 38	5·21	— 37	+ 15	— 35	38	157	104	143	— 14
415	6·8	— 53 47	5·97	— 43	— 19	— 36	41	208	92	143	— 65
418	16·1	— 65 55	4·34	— 40	+ 73	+ 793	790	6	102	140	+ 134
419	17·3	— 55 12	6·36	— 44	— 19	+ 28	34	326	93	140	+ 174
422	28·1	— 65 23	6·45	— 42	— 7	+ 4	8	300	102	138	— 162
423	27·5	— 77 56	3·90	— 35	+ 19	— 258	260	176	111	135	— 41
426	40·2	— 70 12	5·72	— 40	— 55	— 21	59	249	106	134	— 115
428	49·4	— 55 35	4·68	— 48	+ 33	— 37	50	138	97	135	— 3

TABLE I.

Auwers — 52° to — 90°.

Nr.	α 1875	δ 1875	Photom. mag.	b 1875	μ′α	μδ	μ	ψ	λ	χ	χ − ψ
	h m	° ′		°	″	″	″	°	°	°	°
430	21 54·0	− 57 19	4·96	− 48	+ 3·847	− 2·600	4·600	124	98	134	+ 10
435	22 5·9	− 81 3	5·33	− 34	+ 23	− 55	60	157	115	125	− 32
437	9·8	− 60 53	3·07	− 48	87	− 50	100	240	102	130	− 110
438	10·0	− 54 14	5·61	− 52	+ 410	− 665	780	148	97	130	− 18
439	13·9	− 72 52	5·61	− 40	+ 1·260	− 722	1·500	118	110	127	+ 9
443	24·5	− 62 37	4·97	− 48	+ 14	− 28	31	153	105	127	− 26
447	33·1	− 82 2	4·52	− 34	− 68	− 4	68	267	118	120	− 147
451	44·0	− 63 51	6·16	− 49	− 10	− 45	46	193	107	123	− 70
453	45·9	− 70 44	6·44	− 43	− 71	+ 61	94	311	111	121	+ 170
454	53·5	− 53 25	4·30	− 58	− 88	− 7	88	265	103	123	− 142
455	56·5	− 69 30	5·71	− 46	+ 14	+ 67	68	12	111	119	+ 107
460	23 9·4	− 62 41	5·82	− 51	+ 151	− 45	160	107	109	117	+ 19
461	10·2	− 58 55	4·15	− 55	− 63	+ 80	102	322	108	118	+ 156
465	19·6	− 53 25	5·69	− 60	− 17	+ 135	140	353	105	118	+ 125
466	21·8	− 63 48	5·79	− 51	+ 10	− 15	018	146	111	115	− 31
467	25·2	− 78 5	5·94	− 39	− 7	+ 5	9	305	116	109	+ 164
473	37·3	− 71 11	6·13	− 45	+ 227	+ 85	250	70	115	109	+ 39
477	52·4	− 53 27	5·29	− 63	+ 4	+ 50	50	5	111	113	+ 108
478	53·4	− 66 17	4·81	− 51	+ 29	− 38	48	143	115	107	− 36
479	55·2	− 77 45	4·79	− 39	− 93	− 155	170	210	118	101	− 109
480	58·3	− 72 8	5·73	− 45	+ 16	− 27	31	149	117	103	− 46
A	0 12·8	− 89 4	7·39	− 28	+ 10	+ 5	11	63	119	91	+ 28
B	1 44·8	− 85 24	5·81	− 32	+ 23	+ 35	42	33	121	70	+ 37
C	2 39·4	− 86 16	7·93	− 31	− 3	+ 4	5	323	122	54	+ 91
D	3 18·7	− 88 40	8·52	− 29	− 21	− 26	33	219	120	44	− 175
E	4 37·5	− 83 10	6·92	− 31	− 22	+ 22	31	315	126	25	+ 70
F	5 54·5	− 84 51	6·42	− 28	− 18	+ 61	64	344	124	5	+ 21
G	6 12·7	− 85 56	6·95	− 28	− 21	0	21	270	123	0	+ 90
H	7 30·2	− 86 49	6·51	− 27	+ 4	− 2	4	117	122	340	− 137
I	8 11·4	− 88 30	7·92	− 27	− 10	+ 18	21	331	121	329	− 2
K	9 14·4	− 85 10	5·63	− 24	− 120	+ 46	130	294	123	313	+ 19
L	10 38·1	− 85 26	6·9	− 24	− 8	− 25	26	198	121	291	+ 93
M	11 0·1	− 83 55	6·39	− 22	− 71	− 5	71	266	121	285	+ 19
N	12 42·1	− 84 26	5·54	− 22	+ 59	+ 25	64	67	119	259	− 168
O	13 21·0	− 85 8	5·85	− 22	− 89	− 30	94	251	118	251	0
P	14 28·7	− 87 39	6·71	− 25	− 112	− 64	130	240	118	236	− 4
Q	15 14·8	− 84 2	5·87	− 23	+ 134	+ 77	150	60	115	223	+ 163
R	16 14·7	− 86 7	6·21	− 25	+ 5	− 2	5	112	116	209	+ 97
S	17 41·2	− 87 40	5·41	− 27	− 58	− 125	130	205	118	188	− 17
T	18 17·1	− 89 17	5·64	− 27	+ 25	− 4	25	99	119	179	+ 80
U	19 32·9	− 81 40	6·49	− 29	+ 6	+ 2	6	72	112	162	+ 90
V	20 12·2	− 84 49	7·26	− 29	+ 47	+ 33	58	55	115	151	+ 96
W	21 9·3	− 89 26	6·78	− 28	+ 2	− 31	31	176	119	135	− 41
X	22 7·2	− 86 36	5·86	− 30	− 37	+ 70	79	332	118	123	+ 151
IJ	23 8·5	− 88 10	5·75	− 29	+ 10	+ 15	18	34	119	108	+ 74

$f = \frac{1}{CQ}$.

p \ λ	90°	95°	100°	105°	110°	115°	120°	125°	130°	135°	140°	145°	150°	155°	160°	165°	170°	175°	180°
	90°	85°	80°	75°	70°	65°	60°	55°	50°	45°	40°	35°	30°	25°	20°	15°	10°	5°	0°
±																			
0	0·82	0·82	0·82	0·83	0·83	0·84	0·85	0·86	0·88	0·90	0·92	0·94	0·96	0·98	1·01	1·04	1·07	1·10	1·13
5	0·82	0·82	0·82	0·83	0·83	0·84	0·85	0·86	0·88	0·90	0·92	0·94	0·96	0·98	1·01	1·04	1·07	1·10	1·13
10	0·82	0·82	0·82	0·83	0·84	0·85	0·86	0·87	0·88	0·90	0·92	0·94	0·96	0·98	1·01	1·04	1·07	1·10	1·13
15	0·83	0·83	0·83	0·83	0·84	0·85	0·86	0·87	0·89	0·91	0·93	0·95	0·97	0·99	1·01	1·04	1·07	1·10	1·13
20	0·83	0·83	0·84	0·84	0·85	0·86	0·87	0·88	0·89	0·91	0·93	0·95	0·97	0·99	1·02	1·04	1·07	1·10	1·13
25	0·84	0·84	0·85	0·85	0·86	0·86	0·87	0·88	0·90	0·91	0·93	0·95	0·97	0·99	1·02	1·05	1·08	1·10	1·13
30	0·85	0·85	0·86	0·86	0·87	0·87	0·88	0 89	0·91	0·92	0·94	0·96	0·98	1·00	1·03	1·05	1·08	1·10	1·13
35	0·86	0·86	0·87	0·87	0·88	0·88	0·89	0·90	0·92	0·93	0·95	0·97	0·99	1·01	1·03	1·05	1·08	1·11	1·13
40	0·88	0·88	0·88	0·88	0·88	0·89	0·90	0·91	0·92	0·93	0·94	0·96	0·98	1·00	1·02	1·04	1·06	1·09	1·11
45	0·90	0·90	0·90	0·90	0·90	0·91	0·92	0·93	0·94	0·95	0·97	0·99	1·01	1·03	1·05	1·07	1·09	1·11	1·13
50	0·92	0·92	0·92	0·92	0·93	0·93	0·94	0·95	0·96	0·97	0·99	1·00	1·02	1 03	1·05	1·07	1·09	1·11	1·13
55	0·94	0·94	0·94	0·94	0·95	0 95	0·96	0·97	0·98	0·99	1·00	1·01	1·03	1·04	1·06	1·07	1·09	1·11	1 13
60	0·96	0·96	0·96	0·96	0·97	0·97	0·98	0·99	1·00	1·01	1·02	1·03	1·04	1·05	1·07	1·08	1·10	1·11	1·13
65	0·98	0·98	0·98	0·98	0·99	0·99	1·00	1·01	1·02	1·02	1·03	1·04	1·05	1·06	1·08	1·09	1·10	1·11	1·13
70	1·01	1·01	1·01	1·01	1·02	1·02	1·03	1·03	1·04	1·04	1·05	1·06	1·07	1·08	1·09	1·10	1·11	1·12	1·13
75	1·04	1·04	1·04	1·04	1·04	1·04	1·05	1·05	1·06	1·07	1·07	1·07	1·08	1·09	1·10	1·10	1·11	1·12	1·13
80	1·07	1·07	1·07	1·07	1·07	1·08	1·08	1·08	1·09	1·09	1·09	1 09	1·10	1·10	1·11	1·11	1·12	1·12	1·13
85	1·10	1·10	1·10	1·10	1·10	1·10	1·10	1·11	1·11	1·11	1·11	1·11	1·11	1·11	1·11	1·12	1·12	1 12	1·12	1·13
90	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13	1·13
95	1·17	1·17	1·17	1·17	1·16	1·16	1·16	1·16	1·16	1·16	1·16	1·15	1·15	1·15	1·15	1·14	1·14	1·14	1·13
100	1·20	1·20	1·20	1 20	1·19	1·19	1·19	1·19	1·18	1·18	1·17	1·17	1·16	1·16	1·15	1·15	1·14	1·14	1·13
105	1·23	1·23	1·23	1·23	1·22	1·22	1·22	1·21	1 21	1·20	1·19	1·19	1·18	1·18	1·17	1·16	1·15	1·14	1·13
110	1·26	1·26	1·26	1·26	1·25	1·25	1·24	1·24	1·23	1·22	1·21	1·21	1·20	1·19	1·18	1·17	1·15	1·14	1·13
115	1·30	1·30	1·30	1·30	1·29	1·28	1·27	1·27	1·26	1·25	1·23	1·22	1·21	1·20	1·19	1·18	1·16	1·15	1·13
120	1·33	1·33	1·33	1·33	1·32	1·31	1·30	1·29	1·28	1·27	1·25	1·24	1·22	1·21	1·20	1·18			
125	1·36	1·36	1·36	1·36	1·35	1·34	1·33	1·32	1·31	1·29	1·27	1·26	1·24	1·23	1·21	1·19	1·17	1·15	1·13
130	1·38	1·38	1·38	1·38	1·37	1·36	1·35	1·34	1·33	1·31	1·29	1·27	1·25	1·23	1·21	1·19	1·17	1·15	1·13
135	1·41	1·41	1·40	1·40	1·39	1·38	1·37	1·36	1·35	1·33	1·31	1·29	1·27	1·25	1·22	1·20	1·18	1·16	1·13
140	1·43	1·43	1·42	1·42	1·41	1·40	1·39	1·38	1·36	1·35	1·33	1·31	1·28	1·26	1·23	1·21	1·18	1·16	1·13
145	1·46	1·46	1·45	1·44	1·43	1·42	1·41	1·40	1·38	1·36	1·34	1·32	1 29	1·27	1·24	1·21	1 19	1·16	1·13
150	1·48	1·48	1·47	1·46	1·45	1·44	1·42	1·41	1·39	1·37	1·35	1·33	1·30	1·27	1·24	1·22	1·19	1·16	1·13
155	1·50	1·50	1·49	1·48	1·47	1·46	1·44	1·42	1·40	1·39	1·36	1·34	1·31	1·28	1·25	1·22	1·19	1·16	1·13
160	1·51	1·51	1·51	1·50	1·48	1·47	1·45	1·43	1·41	1·39	1·37	1·35	1·32	1·29	1·25	1·22	1·19	1·16	1·13
165	1·53	1·53	1·52	1·51	1·50	1·48	1·46	1·44	1·42	1·40	1·38	1·36	1·33	1·30	1·26	1·23	1·20	1 17	1·13
170	1·54	1·54	1·53	1·52	1·51	1·49	1·47	1·45	1·42	1·40	1·38	1·36	1·33	1·30	1·26	1·23	1·20	1·17	1·13
175	1·55	1.55	1·54	1·53	1·51	1·50	1·48	1·46	1·43	1·41	1·38	1·36	1 33	1·30	1·26	1·23	1·20	1·17	1·13
180	1·55	1.55	1·54	1·53	1·51	1·50	1·48	1·46	1·43	1·41	1·38	1·36	1·33	1·30	1·26	1·23	1·20	1·17	1·13

$1 - \cos^2\delta\cos^2\alpha$.

δ \ α	0h 0m +	0h 30m +	1h 0m +	1h 30m +	2h 0m +	2h 30m +	3h 0m +	3h 30m +	4h 0m +	4h 30m +	5h 0m +	5h 30m +	6h 0m +
	12h 0m +	11h 30m +	11h 0m +	10h 30m +	10h 0m +	9h 30m +	9h 0m +	8h 30m +	8h 0m +	7h 30m +	7h 0m +	6h 30m +	6h 0m +
	12h 0m +	12h 30m +	13h 0m +	13h 30m +	14h 0m +	14h 30m +	15h 0m +	15h 30m +	16h 0m +	16h 30m +	17h 0m +	17h 30m +	18h 0m +
	24h 0m +	23h 30m +	23h 0m +	22h 30m +	22h 0m +	21h 30m +	21h 0m +	20h 30m +	20h 0m +	19h 30m +	19h 0m +	18h 30m +	18h 0m +
±													
0	0·000	0·019	0·067	0·148	0·250	0·371	0·500	0·629	0·750	0·852	0·933	0·081	1·000
5	0·008	0·027	0·074	0·154	0·256	0·376	0·504	0·632	0·752	0·853	0·934	0·981	1·000
10	0·030	0·049	0·095	0·173	0·273	0·390	0·515	0·640	0·758	0·856	0·935	0·981	1·000
15	0·067	0·085	0·130	0·204	0·300	0·413	0·534	0·654	0·767	0·862	0·937	0·982	1·000
20	0·116	0·133	0·175	0·246	0·337	0·444	0·558	0·672	0·779	0·869	0·941	0·983	1·000
25	0·179	0·195	0·234	0·300	0·384	0·483	0·590	0·695	0·795	0·878	0·945	0·984	1·000
30	0·250	0·265	0·300	0·360	0·438	0·530	0·625	0·721	0·813	0·889	0·950	0·986	1·000
35	0·329	0·341	0·374	0·428	0·497	0·578	0·665	0·751	0·832	0·900	0·955	0·987	1·000
40	0·413	0·424	0·452	0·499	0·560	0·630	0·707	0·782	0·853	0·913	0·961	0·988	1·000
45	0·500	0·509	0·534	0·574	0·625	0·686	0·750	0·815	0·875	0·926	0·967	0·990	1·000
50	0·587	0·594	0·615	0·648	0·690	0·740	0·794	0·846	0·897	0·939	0·972	0·992	1·000
55	0·671	0·677	0·693	0·719	0·753	0·793	0·836	0·878	0·918	0·951	0·978	0·993	1·000
60	0·750	0·755	0·767	0·787	0·813	0·842	0·875	0·907	0·938	0·963	0·983	0·995	1·000
65	0·821	0·824	0·833	0·847	0·866	0·887	0·911	0·933	0·955	0·973	0·988	0·997	1·000
70	0·883	0·885	0·891	0·900	0·912	0·926	0·942	0·957	0·971	0·982	0·992	0·997	1·000
75	0·933	0·934	0·937	0·943	0·950	0·958	0·967	0·975	0·983	0·990	0·996	0·998	1·000
80	0·970	0·970	0·972	0·974	0·978	0·981	0·985	0·989	0·993	0·995	0·998	0·999	1·000
85	0·992	0·992	0·993	0·993	0·994	0·995	0·996	0·997	0·998	0·999	0·999	1·000	1·000
90	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000

$1 - \cos^2\delta\sin^2\alpha$.

δ \ α	0h 0m +	0h 30m +	1h 0m +	1h 30m +	2h 0m +	2h 30m +	3h 0m +	3h 30m +	4h 0m +	4h 30m +	5h 0m +	5h 30m +	6h 0m +
	12h 0m +	11h 30m −	11h 0m +	10h 30m +	10h 0m +	9h 30m +	9h 0m +	8h 30m +	8h 0m +	7h 30m +	7h 0m +	6h 30m +	6h 0m +
	12h 0m +	12h 30m +	13h 0m +	13h 30m +	14h 0m +	14h 30m +	15h 0m +	15h 30m +	16h 0m +	16h 30m +	17h 0m +	17h 30m +	18h 0m +
	24h 0m +	23h 30m +	23h 0m +	22h 30m +	22h 0m +	21h 30m +	21h 0m +	20h 30m +	20h 0m +	19h 30m +	19h 0m +	18h 30m +	18h 0m +
±													
0	1·000	0·981	0·933	0·852	0·750	0·629	0·500	0·371	0·250	0·148	0·067	0 019	0·000
5	1·000	0·981	0·934	0·853	0·752	0·632	0·504	0·376	0·256	0·154	0·074	0·027	0·008
10	1·000	0·981	0·935	0·856	0·758	0·640	0·515	0·390	0·273	0·173	0·095	0·049	0·030
15	1·000	0·982	0·937	0·862	0·767	0·654	0·534	0·413	0·300	0·204	0·130	0·085	0·067
20	1·000	0·983	0·941	0 869	0·779	0·672	0·558	0·444	0·337	0·246	0 175	0·133	0·116
25	1·000	0·984	0·945	0·878	0·795	0·695	0·590	0·483	0·384	0·300	0·234	0·195	0·179
30	1·000	0·986	0·950	0·889	0·813	0·721	0·625	0·530	0·438	0·360	0·300	0·265	0·250
35	1·000	0·987	0·955	0·900	0·832	0·751	0·665	0·578	0·497	0·428	0·374	0·341	0·329
40	1·000	0·988	0·961	0·913	0·853	0·782	0·707	0·630	0·560	0·499	0·452	0·424	0·413
45	1·000	0·990	0·967	0·926	0·875	0·815	0·750	0·686	0·625	0·574	0·534	0 509	0 500
50	1·000	0·992	0·972	0·939	0·897	0·846	0·794	0·740	0·690	0·648	0·615	0·594	0·587
55	1·000	0·993	0·978	0·951	0·918	0·878	0·836	0·793	0·753	0·719	0·693	0 677	0·671
60	1·000	0·995	0·983	0·963	0·938	0·907	0·875	0·842	0·813	0·787	0·767	0·755	0·750
65	1·000	0·997	0·988	0·973	0·955	0·933	0·911	0·887	0·866	0·847	0·833	0·824	0·821
70	1·000	0·997	0·992	0·982	0·971	0·957	0 942	0·926	0·912	0·900	0·891	0·885	0·883
75	1·000	0·998	0·996	0·990	0·983	0·975	0·967	0·958	0·950	0·943	0·937	0·934	0·933
80	1·000	0·999	0·998	0·995	0·993	0·989	0·985	0·981	0·978	0·974	0·972	0·970	0·970
85	1·000	1·000	0·999	0·999	0·998	0·997	0·996	0·995	0·994	0·993	0·993	0·992	0·992
90	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000

$\cos^2\delta\sin\alpha\cos\alpha$.

δ \ α	0h 0m +	0h 30m +	1h 0m +	1h 30m +	2h 0m +	2h 30m +	3h 0m +	3h 30m +	4h 0m +	4h 30m +	5h 0m +	5h 30m +	6h 0m +
	12h 0m −	11h 30m −	11h 0m −	10h 30m −	10h 0m −	9h 30m −	9h 0m −	8h 30m −	8h 0m −	7h 30m −	7h 0m −	6h 30m −	6h 0m −
	12h 0m +	12h 30m +	13h 0m +	13h 30m +	14h 0m +	14h 30m +	15h 0m +	15h 30m +	16h 0m +	16h 30m +	17h 0m +	17h 30m +	18h 0m +
	24h 0m −	23h 30m −	23h 0m −	22h 30m −	22h 0m −	21h 30m −	21h 0m −	20h 30m −	20h 0m −	19h 30m −	19h 0m −	18h 30m −	18h 0m −
±													
0	0·000	0·129	0·250	0·352	0·433	0·481	0·500	0·481	0·433	0·352	0·250	0·129	0·000
5	0·000	0·128	0·248	0·349	0·430	0·477	0·496	0·477	0·430	0·349	0·248	0·128	0·000
10	0·000	0·125	0·242	0·341	0·420	0·467	0·485	0·467	0·420	0·341	0·242	0·125	0·000
15	0·000	0·120	0·233	0·328	0·404	0·449	0·466	0·449	0·404	0·328	0·233	0·120	0·000
20	0·000	0·114	0·221	0·311	0·383	0·425	0·442	0·425	0·383	0·311	0·221	0·114	0·000
25	0·000	0·105	0·205	0·289	0·355	0·395	0·410	0·395	0·355	0·289	0·205	0·105	0·000
30	0·000	0·096	0·187	0·264	0·325	0·361	0·375	0·361	0·325	0·264	0·187	0·096	0·000
35	0·000	0·086	0·168	0·236	0·291	0·323	0·335	0·323	0·291	0·236	0·168	0·086	0·000
40	0·000	0·075	0·147	0·206	0·254	0·282	0·293	0·282	0·254	0·206	0·147	0·075	0·000
45	0·000	0·064	0·125	0·175	0·216	0·240	0·250	0·240	0·216	0·175	0·125	0·064	0·000
50	0·000	0·053	0·103	0·145	0·179	0·199	0·206	0·199	0·179	0·145	0·103	0·053	0·000
55	0·000	0·042	0·082	0·116	0·142	0·158	0·164	0·158	0·142	0·116	0·082	0·042	0·000
60	0·000	0·032	0·062	0·088	0·108	0·120	0·125	0·120	0·108	0·088	0·062	0·032	0·000
65	0·000	0·023	0·045	0·063	0·078	0·086	0·089	0·086	0·078	0·063	0·045	0·023	0·000
70	0·000	0·015	0·029	0·041	0·051	0·056	0·058	0·056	0·051	0·041	0·029	0·015	0·000
75	0·000	0·008	0·017	0·024	0·029	0·032	0·033	0·032	0·029	0·024	0·017	0·008	0·000
80	0·000	0·004	0·007	0·010	0·013	0·014	0·015	0·014	0·013	0·010	0·007	0·004	0·000
85	0·000	0·001	0·002	0·003	0·003	0·004	0·004	0·004	0·003	0·003	0·002	0·001	0·000
90	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000

$\cos\delta\sin\delta\cos\alpha$.

δ \ α	0h 0m ±	0h 30m ±	1h 0m ±	1h 30m ±	2h 0m ±	2h 30m ±	3h 0m ±	3h 30m ±	4h 0m ±	4h 30m ±	5h 0m ±	5h 30m ±	6h 0m ±
	12h 0m ∓	11h 30m ∓	11h 0m ∓	10h 30m ∓	10h 0m ∓	9h 30m ∓	9h 0m ∓	8h 30m ∓	8h 0m ∓	7h 30m ∓	7h 0m ∓	6h 30m ∓	6h 0m ∓
	12h 0m ∓	12h 30m ∓	13h 0m ∓	13h 30m ∓	14h 0m ∓	14h 30m ∓	15h 0m ∓	15h 30m ∓	16h 0m ∓	16h 30m ∓	17h 0m ∓	17h 30m ∓	18h 0m ∓
	24h 0m ±	23h 30m ±	23h 0m ±	22h 30m ±	22h 0m ±	21h 30m ±	21h 0m ±	20h 30m ±	20h 0m ±	19h 30m ±	19h 0m ±	18h 30m ±	18h 0m ±
±													
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
5	0·087	0·087	0·084	0·081	0·075	0·069	0·062	0·053	0·043	0·033	0·023	0·012	0·000
10	0·171	0·169	0·165	0·158	0·148	0·136	0·121	0·104	0·085	0·065	0·044	0·023	0·000
15	0·250	0·248	0·241	0·231	0·216	0·198	0·177	0·152	0·125	0·096	0·065	0·033	0·000
20	0·321	0·318	0·310	0·297	0·278	0·255	0·227	0·195	0·160	0·123	0·083	0·042	0·000
25	0·383	0·379	0·370	0·354	0·332	0·304	0·271	0·233	0·191	0·147	0·099	0·050	0·000
30	0·433	0·429	0·418	0·400	0·375	0·344	0·306	0·264	0·216	0·166	0·112	0·057	0·000
35	0·470	0·466	0·454	0·434	0·407	0·373	0·332	0·286	0·235	0·180	0·122	0·062	0·000
40	0·493	0·489	0·476	0·455	0·427	0·391	0·349	0·300	0·246	0·189	0·128	0·065	0·000
45	0·500	0·495	0·483	0·462	0·433	0·396	0·353	0·304	0·250	0·191	0·129	0 065	0·000
50	0·493	0·489	0·476	0·455	0·427	0·391	0·349	0·300	0·246	0·189	0·128	0·065	0·000
55	0·470	0·466	0·454	0·434	0·407	0·373	0·332	0·286	0·235	0·180	0·122	0·062	0·000
60	0·433	0·429	0·418	0·400	0·375	0·344	0·306	0·264	0·216	0·166	0·112	0·057	0·000
65	0·383	0·379	0·370	0·354	0·332	0·304	0·271	0·233	0·191	0·147	0·099	0·050	0·000
70	0·321	0·318	0·310	0·297	0·278	0·255	0·227	0·195	0·160	0·123	0·083	0·042	0·000
75	0·250	0·248	0·241	0·231	0·216	0·198	0·177	0·152	0·125	0·096	0·065	0·033	0·000
80	0·171	0·169	0·165	0·158	0·148	0·136	0·121	0·104	0·085	0·065	0·044	0·023	0·000
85	0·087	0·087	0·084	0·081	0·075	0·069	0·062	0·053	0·043	0·033	0·023	0·012	0·000
90	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000

OPCARD 101 V3

TABLE III.

$\cos\delta \sin\delta \sin\alpha$.

α / δ	$0^h\,0^m$ ±	$0^h\,30^m$ ±	$1^h\,0^m$ ±	$1^h\,30^m$ ±	$2^h\,0^m$ ±	$2^h\,30^m$ ±	$3^h\,0^m$ ±	$3^h\,30^m$ ±	$4^h\,0^m$ ±	$4^h\,30^m$ ±	$5^h\,0^m$ ±	$5^h\,30^m$ ±	$6^h\,0^m$ ±
	$12^h\,0^m$ ±	$11^h\,32^m$ ±	$11^h\,0^m$ ±	$10^h\,30^m$ ±	$10^h\,0^m$ ±	$9^h\,30^m$ ±	$9^h\,0^m$ ±	$8^h\,30^m$ ±	$8^h\,0^m$ ±	$7^h\,30^m$ ±	$7^h\,0^m$ ±	$6^h\,30^m$ ±	$6^h\,0^m$ ±
	$12^h\,0^m$ ∓	$12^h\,30^m$ ∓	$13^h\,0^m$ ∓	$13^h\,30^m$ ∓	$14^h\,0^m$ ∓	$14^h\,30^m$ ∓	$15^h\,0^m$ ∓	$15^h\,30^m$ ∓	$16^h\,0^m$ ∓	$16^h\,30^m$ ∓	$17^h\,0^m$ ∓	$17^h\,30^m$ ∓	$18^h\,0^m$ ∓
	$24^h\,0^m$ ∓	$23^h\,30^m$ ∓	$23^h\,0^m$ ∓	$22^h\,30^m$ ∓	$22^h\,0^m$ ∓	$21^h\,30^m$ ∓	$21^h\,0^m$ ∓	$20^h\,30^m$ ∓	$20^h\,0^m$ ∓	$19^h\,30^m$ ∓	$19^h\,0^m$ ∓	$18^h\,30^m$ ∓	$18^h\,0^m$ ∓
±													
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
5	0·000	0·012	0·023	0·033	0·043	0·053	0·062	0·069	0·075	0·081	0·084	0·087	0·087
10	0·000	0·023	0·044	0·065	0·085	0·104	0·121	0·136	0·148	0·158	0·165	0·169	0·171
15	0·000	0·033	0·065	0·096	0·125	0·152	0·177	0·198	0·216	0·231	0·241	0·248	0·250
20	0·000	0·042	0·083	0·123	0·160	0·195	0·227	0·255	0·278	0·297	0·310	0·318	0·321
25	0·000	0·050	0·099	0·147	0·191	0·233	0·271	0·304	0·332	0·354	0·370	0·379	0·383
30	0·000	0·057	0·112	0·166	0·216	0·264	0·306	0·344	0·375	0·400	0·418	0·429	0·433
35	0·000	0·062	0·122	0·180	0·235	0·286	0·332	0·373	0·407	0·434	0·454	0·466	0·470
40	0·000	0·065	0·128	0·189	0·246	0·300	0·349	0·391	0·427	0·455	0·476	0·489	0·493
45	0·000	0·065	0·129	0·191	0·250	0·304	0·353	0·396	0·433	0·462	0·483	0·495	0·500
50	0·000	0·065	0·128	0·189	0·246	0·300	0·349	0·391	0·427	0·455	0·476	0·489	0·493
55	0·000	0·062	0·122	0·180	0·235	0·286	0·332	0·373	0·407	0·434	0·454	0·466	0·470
60	0·000	0·057	0·112	0·166	0·160	0·264	0·306	0·344	0·375	0·400	0·418	0·429	0·433
65	0·000	0·050	0·099	0·147	0·191	0·233	0·271	0·304	0·332	0·354	0·370	0·379	0·383
70	0·000	0·042	0·083	0·123	0·160	0·195	0·227	0·255	0·278	0·297	0·310	0·318	0·321
75	0·000	0·033	0·065	0·096	0·125	0·152	0·177	0·198	0·216	0·231	0·241	0·248	0·250
80	0·000	0·023	0·044	9·065	0·085	0·104	0·121	0·136	0·148	0·158	0·165	0·169	0·171
85	0·000	0·012	0·023	0·033	0·043	0·053	0·062	0·069	0·075	0·081	0·084	0·087	0·087
90	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000

$\sin\delta \cos\alpha$.

α / δ	$0^h\,0^m$ ±	$0^h\,30^m$ ±	$1^h\,0^m$ ±	$1^h\,30^m$ ±	$2^h\,0^m$ ±	$2^h\,30^m$ ±	$3^h\,0^m$ ±	$3^h\,30^m$ ±	$4^h\,0^m$ ±	$4^h\,30^m$ ±	$5^h\,0^m$ ±	$5^h\,30^m$ ±	$6^h\,0^m$ ±
	$12^h\,0^m$ ∓	$11^h\,30^m$ ∓	$11^h\,0^m$ ∓	$10^h\,30^m$ ∓	$10^h\,0^m$ ∓	$9^h\,30^m$ ∓	$9^h\,0^m$ ∓	$8^h\,30^m$ ∓	$8^h\,0^m$ ∓	$7^h\,30^m$ ∓	$7^h\,0^m$ ∓	$6^h\,30^m$ ∓	$6^h\,0^m$ ∓
	$12^h\,0^m$ ∓	$12^h\,30^m$ ∓	$13^h\,0^m$ ∓	$13^h\,30^m$ ∓	$14^h\,0^m$ ∓	$14^h\,30^m$ ∓	$15^h\,0^m$ ∓	$15^h\,30^m$ ∓	$16^h\,0^m$ ∓	$16^h\,30^m$ ∓	$17^h\,0^m$ ∓	$17^h\,30^m$ ∓	$18^h\,0^m$ ∓
	$24^h\,0^m$ ±	$23^h\,30^m$ ±	$23^h\,0^m$ ±	$22^h\,30^m$ ±	$22^h\,0^m$ ±	$21^h\,30^m$ ±	$21^h\,0^m$ ±	$20^h\,30^m$ ±	$20^h\,0^m$ ±	$19^h\,30^m$ ±	$19^h\,0^m$ ±	$18^h\,30^m$ ±	$18^h\,0^m$ ±
±													
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
5	0·087	0·087	0·084	0·081	0·075	0·069	0·062	0·053	0·043	0·034	0·023	0·012	0·000
10	0·174	0·172	0·168	0·161	0·151	0·138	0·123	0·106	0·087	0·067	0·045	0·023	0·000
15	0·259	0·257	0·250	0·239	0·224	0·205	0·183	0·158	0·129	0·100	0·067	0·034	0·000
20	0·342	0·340	0·330	0·316	0·296	0·271	0.242	0·208	0·171	0·131	0·089	0·045	0·000
25	0·423	0·419	0·409	0·391	0·366	0·335	0·299	0·258	0·211	0·162	0·110	0·056	0·000
30	0·500	0·495	0·483	0·462	0·433	0·396	0·353	0·304	0·250	0·191	0·129	0·065	0·000
35	0·574	0·569	0·554	0·530	0·497	0·455	0·406	0·349	0·287	0·220	0·149	0·075	0·000
40	0·643	0·637	0·621	0·594	0·570	0·510	0·455	0·391	0·321	0·246	0·167	0·084	0·000
45	0·707	0·700	0·683	0·653	0·612	0·561	0·500	0·431	0·353	0·271	0·183	0·093	0·000
50	0·766	0·759	0·740	0·707	0·663	0·607	0·552	0·467	0·383	0·293	0·198	0·100	0·000
55	0·819	0·812	0·791	0·756	0·709	0·649	0·579	0·499	0·409	0·313	0·212	0·107	0·000
60	0·866	0·858	0·837	0·799	0·750	0·686	0·612	0·527	0·433	0·331	0·224	0·113	0·000
65	0·906	0·897	0·875	0·837	0·785	0·718	0·641	0·552	0·453	0·347	0·235	0·118	0·000
70	0·940	0·931	0·908	0·868	0·814	0·745	0·655	0·572	0·470	0·360	0·243	0·123	0·000
75	0·966	0·957	0·933	0·892	0·837	0·766	0·683	0·588	0·483	0·370	0·250	0·126	0·000
80	0·985	0·976	0·952	0·909	0·853	0·781	0·696	0·599	0·492	0·377	0·255	0·128	0·000
85	0·996	0·987	0·962	0·919	0·863	0·790	0·704	0·606	0·498	0·381	0·258	0·130	0·000
90	1·000	0·991	0·966	0·923	0·866	0·793	0·707	0·609	0·500	0·382	0·259	0·131	0·000

$\sin\delta \sin\alpha$.

α / δ	$0^h\,0^m$ ±	$0^h\,30^m$ ±	$1^h\,0^m$ ±	$1^h\,30^m$ ±	$2^h\,0^m$ ±	$2^h\,30^m$ ±	$3^h\,0^m$ ±	$3^h\,30^m$ ±	$4^h\,0^m$ ±	$4^h\,30^m$ ±	$5^h\,0^m$ ±	$5^h\,30^m$ ±	$6^h\,0^m$ ±
	$12^h\,0^m$ ±	$11^h\,30^m$ ±	$11^h\,0^m$ ±	$10^h\,30^m$ ±	$10^h\,0^m$ ±	$9^h\,30^m$ ±	$9^h\,0^m$ ±	$8^h\,30^m$ ±	$8^h\,0^m$ ±	$7^h\,30^m$ ±	$7^h\,0^m$ ±	$6^h\,30^m$ ±	$6^h\,0^m$ ±
	$12^h\,0^m$ ∓	$12^h\,30^m$ ∓	$13^h\,0^m$ ∓	$13^h\,30^m$ ∓	$14^h\,0^m$ ∓	$14^h\,30^m$ ∓	$15^h\,0^m$ ∓	$15^h\,30^m$ ∓	$16^h\,0^m$ ∓	$16^h\,30^m$ ∓	$17^h\,0^m$ ∓	$17^h\,30^m$ ∓	$18^h\,0^m$ ∓
	$24^h\,0^m$ ∓	$23^h\,30^m$ ∓	$23^h\,0^m$ ∓	$22^h\,30^m$ ∓	$22^h\,0^m$ ∓	$21^h\,30^m$ ∓	$21^h\,0^m$ ∓	$20^h\,30^m$ ∓	$20^h\,0^m$ ∓	$19^h\,30^m$ ∓	$19^h\,0^m$ ∓	$18^h\,30^m$ ∓	$18^h\,0^m$ ∓
±													
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
5	0·000	0·012	0·023	0·034	0·043	0·053	0·062	0·069	0·075	0·081	0·084	0·087	0·087
10	0·000	0·023	0·045	0·067	0·087	0·106	0·123	0,138	0·151	0·161	0·168	0·172	0·174
15	0·000	0·034	0·067	0·100	0·129	0·158	0·183	0·205	0·224	0·239	0·250	0.257	0·259
20	0·000	0·045	0·089	0·131	0·171	0·208	0·242	0·271	0·296	0·316	0·330	0·340	0·342
25	0·000	0·056	0·110	0·162	0·211	0·258	0·299	0·335	0·366	0·391	0·409	0·419	0·423
30	0·000	0·065	0·129	0·191	0·250	0·304	0·353	0·396	0·433	0·462	0·483	0·495	0·500
35	0·000	0·075	0·149	0·220	0·287	0·349	0·406	0·455	0·497	0·530	0·554	0.569	0·574
40	0·000	0·084	0·167	0·246	0·321	0·391	0·455	0·510	0·557	0·594	0·621	0·637	0·643
45	0·000	0·093	0·183	0·271	0·353	0·431	0·500	0·561	0·612	0·653	0·683	0·700	0·707
50	0·000	0·100	0·198	0·293	0·383	0·467	0·542	0·607	0·663	0·707	0·740	0·759	0·766
55	0·000	0·107	0·212	0·313	0·409	0·499	0·579	6·649	0·709	0·756	0·791	0·812	0·819
60	0·000	0·113	0·224	0·331	0·433	0·527	0·612	0·686	0·750	0·799	0·837	0·858	0·866
65	0·000	0·118	0·235	0·347	0·453	0·552	0·641	0·718	0·785	0·837	0·875	0·897	0·906
70	0·000	0·123	0·243	0·360	0·470	0·572	0·665	0·745	0·814	0·868	0·908	0·931	0·940
75	0·000	0·126	0·250	0·370	0·483	0·588	0·683	0·766	0·837	0·892	0·933	0·957	0·966
80	0·000	0·128	0·255	0·377	0·492	0·599	0·696	0·781	0·853	0·909	0·952	0·976	0·985
85	0·000	0·130	0·258	0·381	0·498	0·606	0·704	0·790	6·863	0·919	0·962	0·987	0·996
90	0·000	0·131	0·259	0·382	0·500	0·609	0·707	0·793	0·866	0·923	0·966	0·991	1·000

Gal. lat.	A	B	C	D	E	F	P	Q	R	N
Bradley + 90° to + 52°.										
— 20° to + 20°	+ 135·50	+ 140·71	+ 31·79	0·00	+ 42·03	+ 2·65	— 416	— 2586	+ 439	154
+ 20 to + 40	+ 91.65	+ 83·70	+ 12·64	— 2·26	— 6·42	— 3·11	— 338	— 1772	+ 80	94
+ 40 to + 90	+ 38·44	+ 46·51	+ 13·04	— 1·62	— 18·95	+ 3·43	+ 399	— 867	+ 322	49
All stars together	+ 265·59	+ 270·92	+ 57·47	— 3·88	+ 16·66	+ 2·97	— 355	— 5225	+ 841	297
Bradley + 52° to — 20°.										
— 90° to — 40°	+ 67·11	+ 279·86	+ 320·94	+ 17·06	— 1·52	+ 5·69	— 179	— 5215	+ 4833	334
— 40 to — 20	+ 180·42	+ 184·22	+ 283·30	+ 13·42	+ 53·06	+ 19·41	— 608	— 4761	+ 5660	324
— 20 to + 20	+ 586·64	+ 229·49	+ 539·77	— 68·29	+ 77·36	+ 17·17	— 2665	— 6772	+ 7675	678
+ 20 to + 40	+ 208·98	+ 123·30	+ 251·68	— 20·34	— 13·36	+ 12·36	+ 110	— 4224	+ 4387	292
+ 40 to + 90	+ 177·06	+ 389·42	+ 407·44	+ 15·63	— 93·45	— 3·53	+ 1738	— 12010	+ 6376	487
All stars together	+ 1220·21	+ 1206·29	+ 1803·13	— 42·52	+ 22·09	+ 51·10	— 1604	— 32982	+ 28931	2115
Bradley South. of — 20°.										
— 90° to — 40°	+ 15·33	+ 24·24	+ 28·42	— 5·02	— 9·90	+ 2·07	+ 68	— 438	+ 287	34
— 40 to — 20	+ 8·68	+ 3·02	+ 8·30	— 1·84	— 1·39	+ 1·36	— 46	— 92	+ 68	10
— 20 to + 20	+ 52·51	+ 12·60	+ 46·87	— 2·49	+ 2·50	+ 9·13	+ 30	— 634	+ 603	56
+ 20 to + 40	+ 17·56	+ 29·12	+ 31·33	+ 7·68	+ 12·50	+ 5·68	+ 23	— 951	+ 849	39
All stars together	+ 94·08	+ 68·98	+ 114·92	— 1·67	+ 3·71	+ 18·24	+ 75	— 2115	+ 1807	139
Newcomb — 20° to — 40°.										
— 90° to — 40°	+ 18·65	+ 27·16	+ 28·19	+ 1·09	— 11·93	— 1·65	+ 118	— 685	+ 347	37
— 40 to — 20	+ 15·01	+ 5·74	+ 13·25	— 2·02	— 1·90	— 1·08	+ 36	— 18	+ 369	17
— 20 to + 20	+ 39·58	+ 14·97	+ 33·44	— 2·02	+ 3·98	+ 6·01	+ 86	— 774	+ 807	44
+ 20 to + 40	+ 6·10	+ 13·52	+ 12·38	+ 2·32	+ 5·79	+ 1·47	— 184	— 285	+ 496	16
All stars together	+ 79·34	+ 61·39	+ 87·26	— 0·63	— 4·06	+ 4·75	+ 56	— 1762	+ 2019	114
Taylor — 20° to — 35°.										
— 90° to — 40°	+ 26·12	+ 45·67	+ 48·20	— 9·69	— 19·15	+ 4·77	+ 76	— 343	+ 1040	60
— 40 to — 20	+ 15·92	+ 8·73	+ 17·34	— 6·27	— 4·07	+ 4·34	— 283	— 274	+ 148	21
— 20 to + 20	+ 66·35	+ 20·47	+ 57·16	— 2·89	+ 4·18	+ 11·91	+ 250	— 651	+ 1601	72
+ 20 to + 40	+ 17·03	+ 35·42	+ 33·55	+ 5·01	+ 15·35	+ 3·18	— 433	— 1054	+ 720	43
+ 40 to + 90	+ 0·25	+ 2·00	+ 1·76	+ 0·02	+ 0·65	+ 0·01	— 24	+ 45	+ 61	2
All stars together	+ 125·67	+ 112·29	+ 158·01	— 13·82	— 3·04	+ 24·21	— 414	— 2277	+ 3570	198
Newcomb — 40° to — 52°.										
— 90° to — 40°	+ 4·77	+ 5·49	+ 3·74	+ 0·48	— 2·61	— 0·83	+ 34	— 105	+ 99	7
— 40 to — 20	+ 2·95	+ 1·52	+ 1·54	— 0·16	— 0·17	— 0·67	— 4	— 59	— 15	3
— 20 to + 20	+ 21·65	+ 20·22	+ 14·14	+ 1·85	+ 8·40	+ 3·67	— 133	— 635	+ 390	28
All stars together	+ 29·37	+ 27·23	+ 19·42	+ 2·17	+ 5·62	+ 2·17	— 103	— 799	+ 474	38

Gal. lat.	A	B	C	D	E	F	P	Q	R	N
Gill — 40° to — 52°.										
— 90° to — 40°	+ 114·21	+ 145·80	+ 84·00	+ 10·65	— 69·01	— 18·69	+ 2376	— 3289	+ 1747	172
— 40 to — 20	+ 68·32	+ 41·39	+ 34·29	— 4·97	— 7·53	— 5·47	— 19	— 765	+ 436	72
— 20 to + 20	+ 184·69	+ 178·57	+ 114·76	— 7·89	+ 72·79	— 3·97	— 2466	— 5232	+ 3490	239
+ 20 to + 40	+ 3·22	+ 6·80	+ 3·99	+ 0·75	+ 3·37	+ 0·68	— 101	— 358	+ 171	7
All stars together	+ 370·44	+ 372·56	+ 237·04	— 1·46	— 0·38	— 27·45	— 210	— 9644	+ 5844	490
Combination Newcomb—Gill.										
— 90° to — 40°	+ 129·33	+ 164·19	+ 95·68	+ 10·08	— 77·79	— 18·35	+ 2524	— 3719	+ 2116	
— 40 to — 20	+ 81·92	+ 50·39	+ 41·69	— 7·49	— 10·87	+ 1·37	— 13	— 1133	+ 669	
— 20 to + 20	+ 216·09	+ 206·00	+ 135·36	— 0·47	+ 82·74	+ 14·33	— 2381	— 6430	+ 4311	
+ 20 to + 40	+ 3·22	+ 6·80	+ 3·99	+ 0·75	+ 3·37	+ 0·68	— 101	— 358	+ 171	
All stars together	+ 430·56	+ 427·38	+ 276·72	+ 2·87	— 2·55	— 1·97	+ 29	— 11640	+ 7267	
Auwers — 52° to — 90°.										
— 90° to — 40°	+ 42·72	+ 47·70	+ 11·58	+ 0·76	— 17·05	— 2·85	+ 286	— 921	+ 95	51
— 40 to — 20	+ 84·45	+ 78·02	+ 9·51	— 1·59	— 4·83	+ 0·37	— 192	— 1774	+ 124	86
— 20 to + 20	+ 77·04	+ 79·91	+ 19·04	— 0·05	+ 24·65	+ 3·56	— 648	— 2823	+ 732	88
All stars together	+ 204·21	+ 205·63	+ 40·13	— 0·88	+ 2·77	+ 1·08	— 554	— 5518	+ 951	225

TABLE V.

Bradley.

Corrections depending on the declination.

Decl.	$\Delta\mu_\alpha$	$\Delta\mu_\delta$
Southern	+ 0ˢ·00035	+ 0″·0004
0° to 20°	+ 23	— 17
20° to 40°	+ 16	— 25
40° to 60°	0	— 12
60° to 90°	— 10	+ 55

Corrections depending on the rightascension.

AR	$\Delta\mu_\alpha$	$\Delta\mu_\delta$
0ʰ	+ 0ˢ·00005	+ 0″·0005
1ʰ	+ 8	0
2ʰ	+ 2	+ 1
3ʰ	— 8	0
4ʰ	— 12	— 5
5ʰ	— 13	— 12
6ʰ	— 16	— 9
7ʰ	— 16	— 6
8ʰ	— 11	0
9ʰ	— 6	+ 10
10ʰ	— 6	+ 16
11ʰ	— 8	+ 7
12ʰ	— 15	— 2
13ʰ	— 16	+ 7
14ʰ	— 4	+ 18
15ʰ	+ 6	+ 14
16ʰ	+ 3	+ 7
17ʰ	0	— 1
18ʰ	+ 10	— 4
19ʰ	+ 18	— 2
20ʰ	+ 14	— 2
21ʰ	+ 7	+ 2
22ʰ	— 1	+ 10
23ʰ	— 3	+ 11

TABLE VI.

Gal. lat.	ΔP	ΔQ	ΔR
Bradley.			
— 90° to — 40°	— 8	— 1787	— 19
— 40 to — 20	— 325	— 1019	+ 514
— 20 to + 20	— 1290	— 915	+ 1442
+ 20 to + 40	— 365	+ 486	+ 15
+ 40 to + 90	— 103	+ 702	+ 189
All stars together	— 2091	— 2533	+ 2141
Newcomb — 20° to — 40°.			
— 90° to — 40°	— 40	— 6	— 88
— 40 to — 20	— 7	— 3	— 41
— 20 to + 20	+ 18	+ 32	— 167
+ 20 to + 40	+ 12	+ 3	— 23
All stars together	— 17	+ 26	— 319
Taylor — 20° to — 35°.			
— 90° to — 40°	— 98	+ 27	— 233
— 40 to — 20	— 22	+ 26	— 87
— 20 to + 20	+ 24	+ 77	— 365
+ 20 to + 40	+ 89	+ 19	— 200
+ 40 to + 90	+ 9	0	— 16
All stars together	+ 2	+ 149	— 901
Newcomb — 40° to — 52°.			
— 90° to — 40°	— 15	— 5	— 19
— 40 to — 20	— 1	— 3	— 8
— 20 to + 20	+ 50	+ 24	— 83
All stars together	+ 34	+ 16	— 110
Gill — 40° to — 52°.			
— 90° to — 40°	— 534	— 145	— 639
— 40 to — 20	— 78	— 60	— 377
— 20 to + 20	+ 770	— 42	— 1221
+ 20 to + 40	+ 20	+ 4	— 20
All stars together	+ 178	— 243	— 2257

TABLE VII.

Groups of different galactic latitude.

Gal. lat.	A	B	C	D	E	F	P	Q	R
— 90 to — 40°	+ 190·85	+ 444·92	+ 441·13	+ 16·89	— 73·77	— 4·29	+ 1037	— 10496	+ 6243
— 40 to — 20	+ 287·42	+ 256·38	+ 332·61	+ 4·56	+ 42·42	+ 17·55	— 1176	— 7316	+ 6654
— 20 to + 20	+ 983·33	+ 552·35	+ 747·20	— 75·63	+ 184·02	+ 41·40	— 5424	— 16453	+ 12916
+ 20 to + 40	+ 330·16	+ 261·89	+ 318·41	— 10·98	+ 4·03	+ 17·54	— 868	— 7178	+ 6009
+ 40 to + 90	+ 215·56	+ 436·43	+ 420·92	+ 14·01	— 112·24	— 0·10	+ 2030	— 12164	+ 6898
All groups together	+ 2007·32	+ 1951·97	+ 2260·27	— 51·15	+ 44·46	+ 72·10	— 4401	— 53607	+ 38720

TABLE VIII.

Groups of different type of spectrum.

Type.	A	B	C	D	E	F	P	Q	R
I	+ 741·99	+ 637·10	+ 858·78	— 17·75	+ 36·42	+ 30·96	— 1746	— 18049	+ 13192
II	+ 625·51	+ 695·27	+ 809·01	+ 6·22	+ 10·27	+ 25·07	— 1330	— 19278	+ 14704

TABLE IX.

All stars together (definitive results).

A	B	C	D	E	F	P	Q	R
+ 2404·75	+ 2330·05	+ 2506·39	+ 50·79	— 33·80	— 86·97	— 4525	— 64017	+ 43502

STELLINGEN.

I.

Aan de eigenbewegingen in declinatie volgens Newcomb's fundamentaalcatalogus van de sterren zuidelijk van — 20° declinatie moeten waarschijnlijk positieve systematische correcties worden aangebracht.

II.

Het traagheidsprincipe is geen zuivere ervaringswet.

III.

Het theorema van het arithmetisch gemiddelde moet niet als een axioma worden opgevat.

IV.

De onderstelling, dat bij gasvormige hemellichamen in elkaars nabijheid vloedgolven worden opgewekt aan de naar elkaar toegekeerde en van elkaar afgekeerde zijden is niet algemeen juist.

V.

De spectrographische methode van Küstner ter bepaling van de zonsparallax verdient naast de andere methoden ter bepaling van deze grootheid in aanmerking te komen.

VI.

De hypothese van Pickering ter verklaring van de retrograde beweging van Phoebe uit een oorspronkelijk retrograde omwentelingsbeweging van Saturnus (Harv. Ann. 53) is niet aannemelijk.

VII.

De grondeigenschap, gewoonlijk als axioma in de rekenkunde vooropgesteld, is een bewijsbare eigenschap.

VIII.

Ten onrechte ziet Poincaré in de mogelijkheid van het bestaan van een niet-Euclidische meetkunde een argument tegen de opvatting van de meetkundige axioma's als synthetische oordeelen a priori (la Science et l'Hypothèse pag. 64).

IX.

De bewering van Poincaré, dat het aanvaarden van de meetkundige axioma's slechts berust op een overeenkomst op eenvoudigheidsoverwegingen gebaseerd, is onjuist. (la Science et l'Hypothèse pag. 66.)

X.

De invoering van het begrip differentiaal in overeenstemming met de wijze, waarop Newton de differentiaalrekening baseert, verdient de voorkeur boven die volgens de methode van Leibnitz.

XI.

Door de onderstelling, dat voor electronen met een snelheid grooter dan die van het licht de traagheidswet niet geldt, wordt aan het traagheidsprincipe in theoretisch-mechanischen zin niet te kort gedaan.

XII.

Ten onrechte beweert Lippmann, dat de opvatting van een electrischen stroom als een beweging van electronen eischt, dat aan het begrip absolute beweging een physische beteekenis wordt toegekend.

XIII

De studie van de Erkenntnisstheorie is van belang voor de beoefenaars der natuurwetenschap.

XIV.

Uitgezonderd voor de pharmaceuten zou één algemeen candidaatsexamen in de faculteit der wis- en natuurkunde gewenscht zijn.

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